CAD Algorithms for Physical Design - Placement

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  - Optimal Local Placement
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  - Centre of Mass
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  - QP
  - Density Mitigation
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- Kraftwerk2
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- Timing-Driven Placement
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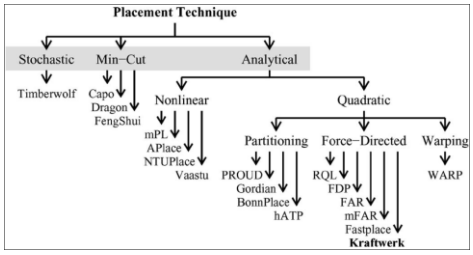
Placement Algorithms

- Combinatorial methods
  - Min-cut placement
  - Cluster growth
- Nondeterministic methods:
  - Simulated annealing (SA)
  - Genetic algorithm (GA)
- Analytical methods:
  - Force-directed placement
  - Quadratic placement (QP)
  - Non-quadratic placement
- Mixed-size placement



Placement Problem

- **Placement Problem:** assign cells to positions on the chip, such that no two cells overlap with each other (legalization) and some cost function (e.g., wirelength) is optimized
  - A major step in physical design that has been studied for 40+ years.

Modern Placement Challenges

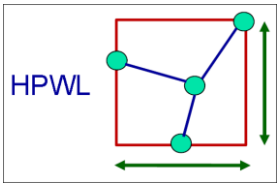
- High complexity
  - Millions of objects to be placed
- Placement constraints
  - Preplaced blocks
  - Chip density, etc.
- **Mixed-size placement**
  - Hundreds/thousands of large macros with millions of small standard cells
- **Datapath placement**
- Regular structure, functional stage alignment
- Many more

Basic Wirelength Models

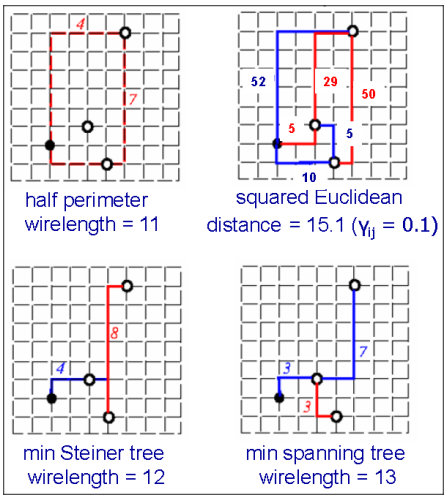
- **Half-perimeter wirelength (HPWL):** Half the perimeter of the bounding rectangle that encloses all the pins of the net to be connected
  - Most widely used approximation!
- **Squared Euclidean distance:** squares of all pairwise terminal distances in a net using a quadratic cost function

$$\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}\left[(x_i - x_j)^2+(y_i - y_j)^2\right]$$

- **Steiner-tree approximation:** Computationally expensive
- **Minimum spanning tree**: Good approximation to Steiner trees

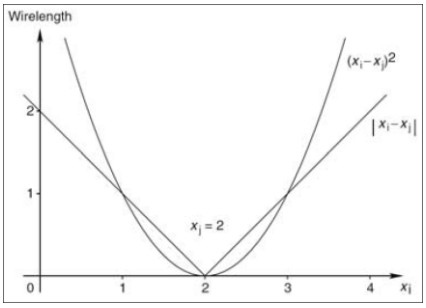


Wirelength Estimation Examples



Quadratic Wirelength vs. Linear Wirelength

- HPWL is convex, but not differentiable
- Quadratic wirelength is a convex, differentiable function
  - easier/faster to minimize
- Both models correlate with each other "to some degree"
  - (might incur large errors if two cells are far away)

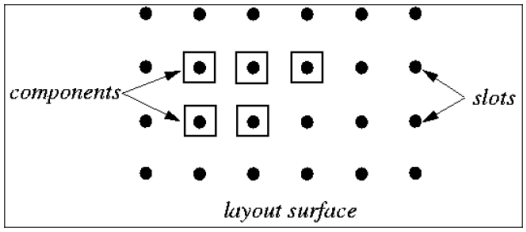


Placement Algorithm Paradigms

- The placement problem is NP-complete.
- Popular placement algorithms:
  - **Constructive algorithms:** once the position of a cell is fixed, it is not modified anymore
    - Cluster growth, min cut, QP, etc.
  - **Iterative algorithms:** intermediate placements are modified in an attempt to improve the cost function.
    - Force-directed method, nonlinear placement, etc
  - **Nondeterministic approaches:** simulated annealing, genetic algorithm, etc.
- May combine multiple elements:
  - Constructive algorithms are used to obtain an **initial placement**
  - The initial placement is followed by **iterative improvement**
  - The results can further be improved by **simulated annealing**

Bottom-up Cluster Growth Placement

- Greedy method: Selects unplaced components and places them in available slots.
  - SELECT: Choose the unplaced component that is most strongly connected to all of the placed components (or most strongly connected to any single placed component)
  - PLACE: Place the selected component at a slot such that a certain "cost" of the partial placement is minimized



C.M. Kyung, J.M. Widder, D.A. Mlynksi, "Adaptive Cluster Growth (ACG): A New Algorithm for Circuit Packing in Rectalinear Region", Proceedings of EDAC, March 1990

Adaptive Cluster Growth

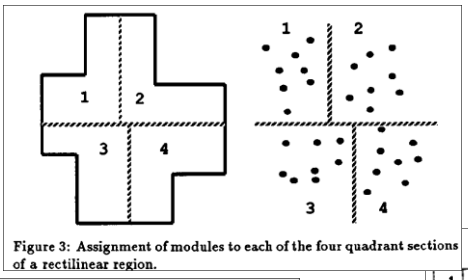


Figure 3: Assignment of modules to each of the four quadrant sections of a rectilinear region.

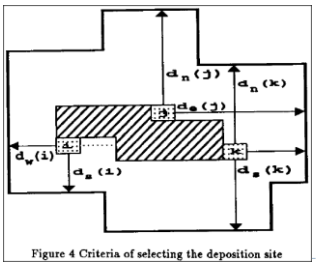
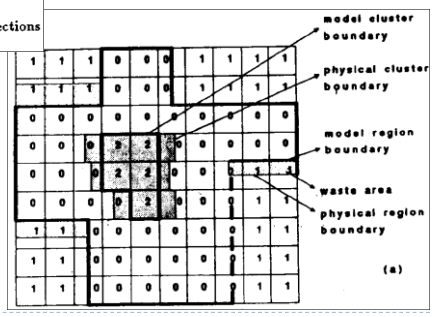


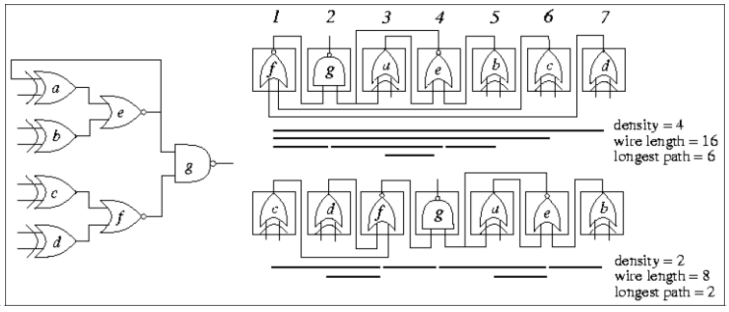
Figure 4 Criteria of selecting the deposition site



The module located closest to the rectilinear polygon is selected as seed. Module cluster boundary, tiles are labelled "2". Outside tiles tagged as "1".
Order criterion: minimum cell distance from deposition site

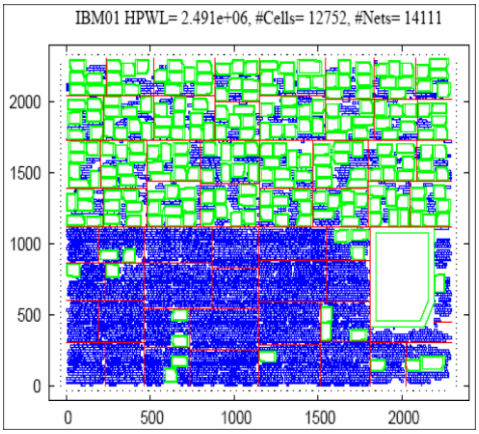
Cluster Growth Example

- # of other terminals connected: *ca*=3, *cb*=1, *cc*=1, *cd* =1, *ce*=4, *cf*=3, and *cg*=3; e has the most connectivity
- Place e in the center, slot 4. *a, b, g* are connected to e, and *cae* = 2, *cbe* = *ceg* = 1; Place *a* next to e (say, slot 3).
- Continue until all cells are placed
- Further improve the placement by swapping gates



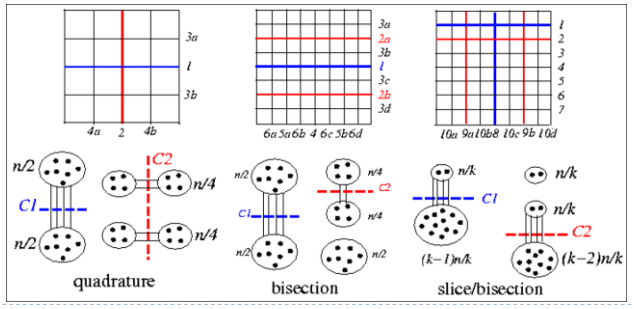
Partitioning-based Placement

- Rely on a good partitioner (e.g., hMetis) and partitionmetric (e.g., exact netweight modeling)
- Pros
  - Is very fast
  - Has very good scalability for handling large-scale designs
- Cons
  - Relatively harder to handle design with large white spaces



Top-Down Min-Cut Placement

- Breuer, "A class of min-cut placement algorithms," DAC-77.
- **Quadrature:** suitable for circuits with high density in the center
- **Bisection:** good for standard-cell placement
- **Slice/Bisection:** good for cells with high interconnection on the periphery



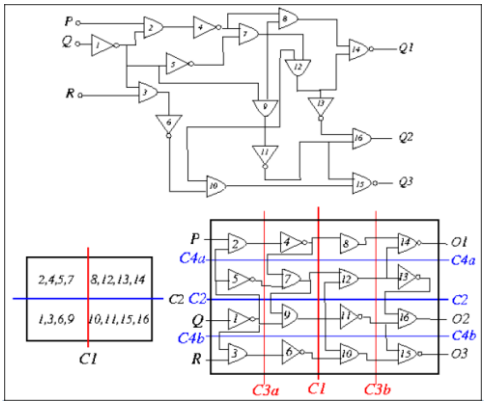
Min-Cut Placement Algorithm

- **Algorithm: Min_Cut_Placement(*N*, *n*, *C*)**

```
/* N: the layout surface */
/* n : # of cells to be placed */
/* n0: # of cells in a slot */
/* C: the connectivity matrix */
1 begin
2 if (n ≤ n0) then PlaceCells(N, n, C)
3 else
4   (N1, N2) = CutSurface(N);
5   (n1, C1), (n2, C2) = Partition(n, C);
6 Call Min_Cut_Placement(N1, n1, C1);
7 Call Min_Cut_Placement(N2, n2, C2);
8 end
```

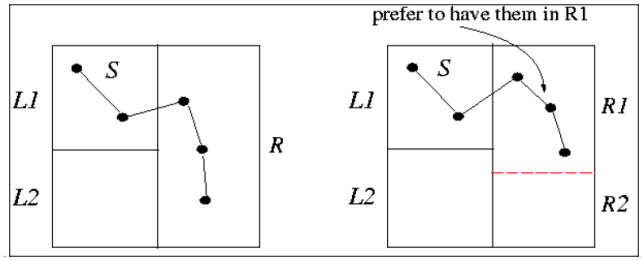
Quadrature Placement Example

- Apply the F-M or K-L heuristic to partition + Quadrature Placement:
  - Cost *C1* = 4, *C2L*= *C2R* = 2, etc.



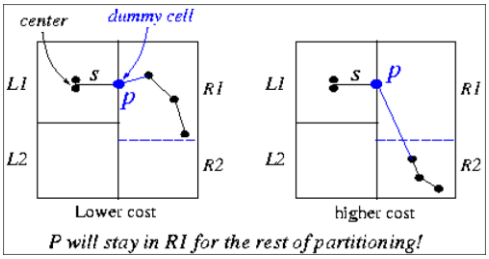
Min-Cut Placement with Terminal Propagation

- Dunlop & Kernighan, "A procedure for placement of standard-cell VLSI circuits," *IEEE TCAD*, Jan. 1985.
- Drawback of the original min-cut placement: Does not consider the positions of terminal pins that enter a region
- What happens if we swap {1, 3, 6, 9} and {2, 4, 5, 7} in the previous example?



Terminal Propagation

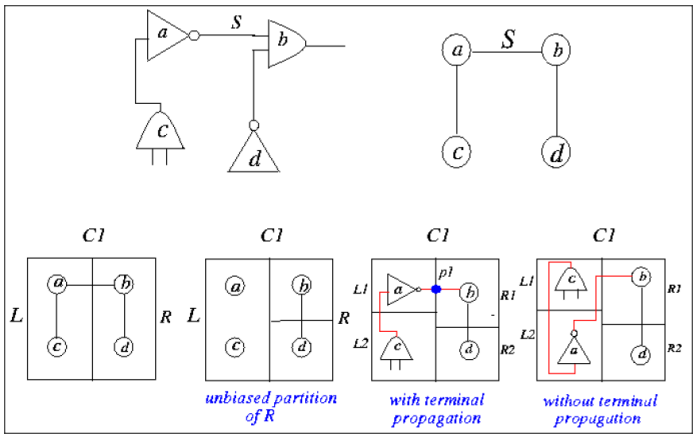
- Should use the fact that s is in L1!



  - minimum WL solution is used to select which of the two partitions go into R1 (up) and R2 (down)

The minimum WL solution is selected to choose which of the two partitions go into R1 (up) and R2 (down)

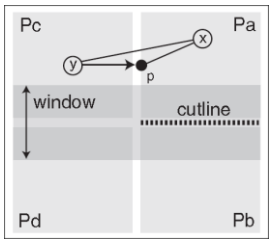
Terminal Propagation Example



Partitioning must be done breadth-first, not depth first

Terminal Propagation in more Detail

- Assume that left half of the chip *L* is already partitioned into {*Pc, Pd*}, $y \in Pc$ is located at the center of *Pc*
- Now partitioning the right half *R* into {*Pa, Pb*}
- For each gate $x \in R$ connected to another gate *y, outside R*, check to see if *y* is too ***closely located*** to the cutline

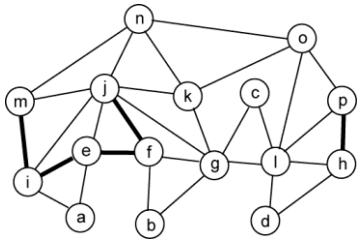


  - **inside the "window"; two parallel lines to the cutline**
  - If *y* is outside the window, "propagate" *y* to *p*
    - where *p* is point **on the boundary of *R***
  - *p* becomes the "propagated terminal" for *y*
  - We then connect *x* and *p* and ignore the *x*-*y* connection
  - During the partitioning refinement, *p* is fixed at this initial location, and acts like an "anchor" that pulls *x* into *Pa* that *p* is locked in
  - This is due to the fact that if *x* is not partitioned together with *p*, the *x*-*p* connection increases the cutsize by one

Min-Cut Placement Example

- Perform quadrature mincut onto 4 × 4 grid
  - Start with vertical cut first

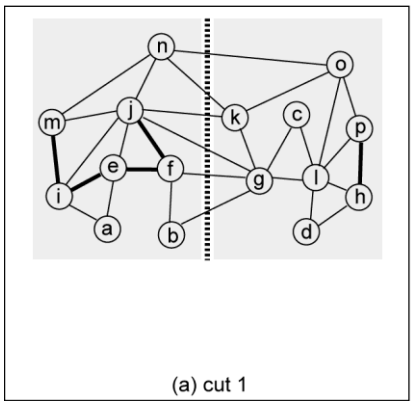
$n_1 = \{e, f\}$
$n_2 = \{a, e, i\}$
$n_3 = \{b, f, g\}$
$n_4 = \{c, g, l\}$
$n_5 = \{d, l, h\}$
$n_6 = \{e, i, j\}$
$n_7 = \{f, j\}$
$n_8 = \{g, j, k\}$
$n_9 = \{l, o, p\}$
$n_{10} = \{h, p\}$
$n_{11} = \{i, m\}$
$n_{12} = \{j, m, n\}$
$n_{13} = \{k, n, o\}$



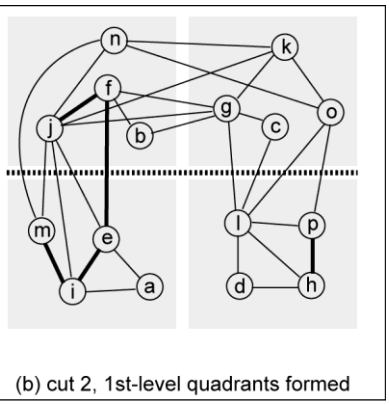
undirected graph model w/ k-clique weighting
thin edges = weight 0.5, thick edges = weight 1

Min-Cut Placement Example – Cuts 1, 2

- First cut has min-cutsize of 3 (not unique)
  - Both cuts 1 and 2 divide the entire chip



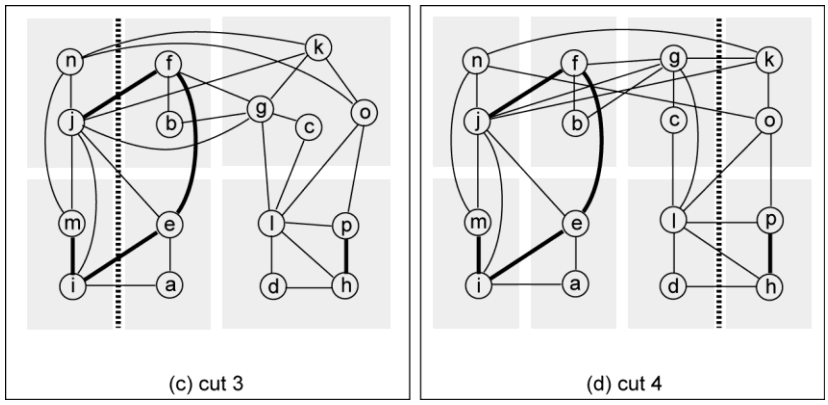
(a) cut 1



(b) cut 2, 1st-level quadrants formed

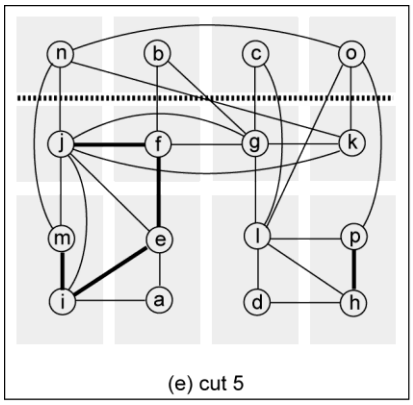
Min-Cut Placement Example – Cuts 3, 4

- Each cut minimizes cutsize
  - Helps reduce overall wirelength

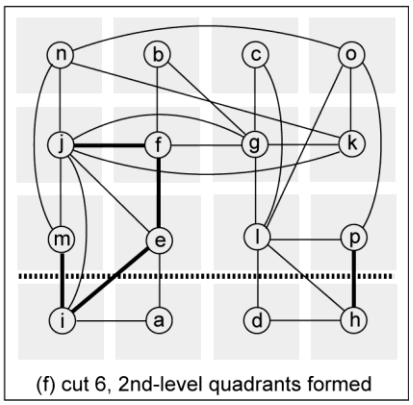


Min-Cut Placement Example – Cuts 5, 6

- 16 partitions generated by 6 cuts
  - HPBB wirelength = 27



(e) cut 5

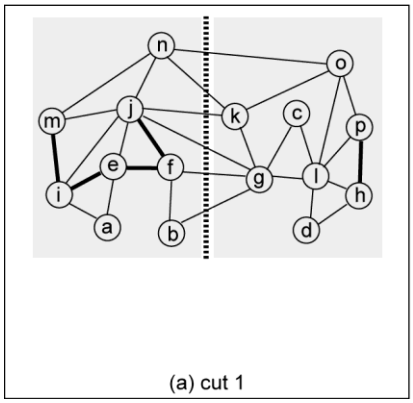


(f) cut 6, 2nd-level quadrants formed

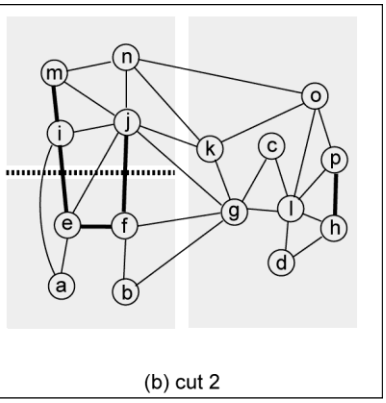
we compute the half-perimeter of the bounding box of the nets as follows: $w(n1) = 1$, $w(n2) = 2$, $w(n3) = 2$, $w(n4) = 2$, $w(n5) = 2$, $w(n6) = 3$, $w(n7) = 1$, $w(n8) = 3$, $w(n9) = 3$, $w(n10) = 1$,
$w(n11) = 1$, $w(n12) = 2$, $w(n13) = 4$. Thus, the total wirelength cost is 27.

Recursive Bisection Example with Terminal Propagation – Cuts 1 and 2

- Start with vertical cut
  - Perform terminal propagation with middle third window



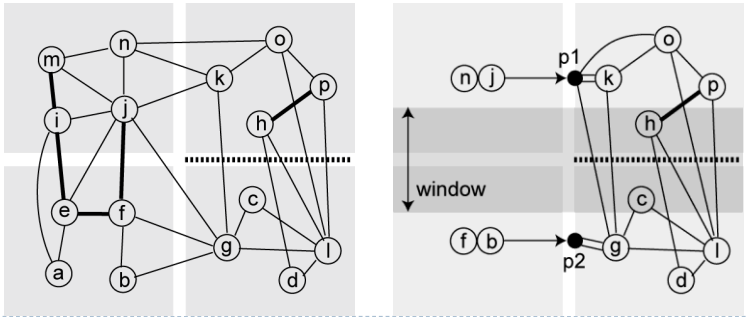
(a) cut 1



(b) cut 2

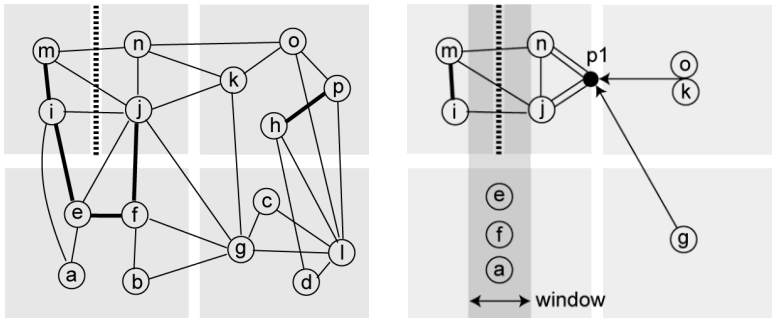
Recursive Bisection Example with Terminal Propagation – Cut 3

- Two terminals are propagated and are "pulling" nodes
  - Node *k* and *o* connect to *n* and *j*: p_1 propagated (outside window)
  - Node *g* connect to *j*, *f* and *b*: p_2 propagated (outside window)
  - Terminal p_1 pulls *k*/*o*/*g* to top partition, and p_2 pulls *g* to bottom



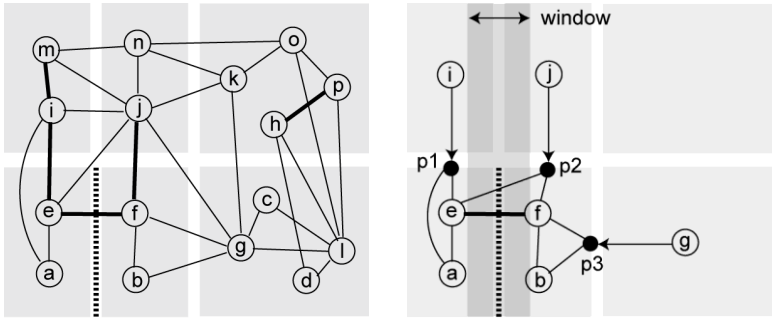
Recursive Bisection Example with Terminal Propagation – Cut 4

- One terminal propagated
  - Node *n* and *j* connect to *o*/*k*/*g*: p_1 propagated
  - Node *i* and *j* connect to *e*/*f*/*a*: no propagation (inside window)
  - Terminal p_1 pulls *n* and *j* to right partition



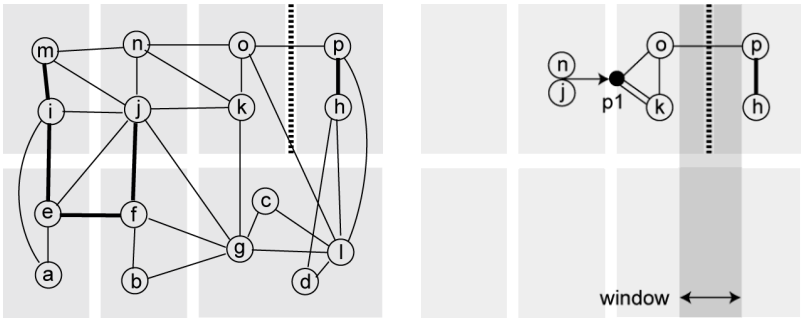
Recursive Bisection Example with Terminal Propagation – Cut 5

- Three terminals propagated
  - Node *i* propagated to p_1, *j* to p_2, and *g* to p_3
  - Terminal p_1 pulls e and *a* to left partition
  - Terminal p_2 and p_3 pull *f*/*b*/e to right partition



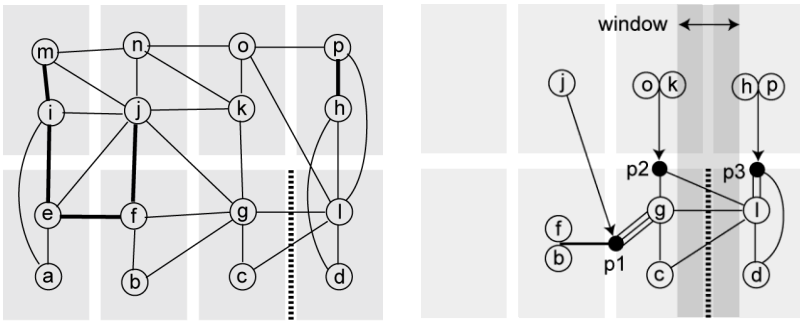
Recursive Bisection Example with Terminal Propagation – Cut 6

- One terminal propagated
  - Node *n* and *j* are propagated to p_1
  - Terminal p_1 pulls *o* and *k* to left partition



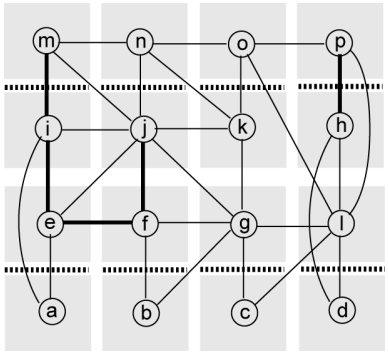
Recursive Bisection Example with Terminal Propagation – Cut 7

- Three terminals propagated
  - Node *j*/*f*/*b* propagated to p_1, *o*/*k* to p_2, and *h*/*p* to p_3
  - Terminal p_1 and p_2 pull *g* and *l* to left partition
  - Terminal p_3 pull *l* and *d* to right partition



Recursive Bisection Example with Terminal Propagation – Cuts 8 to 15

- 16 partitions generated by 15 cuts
  - HPBB wirelength = 23



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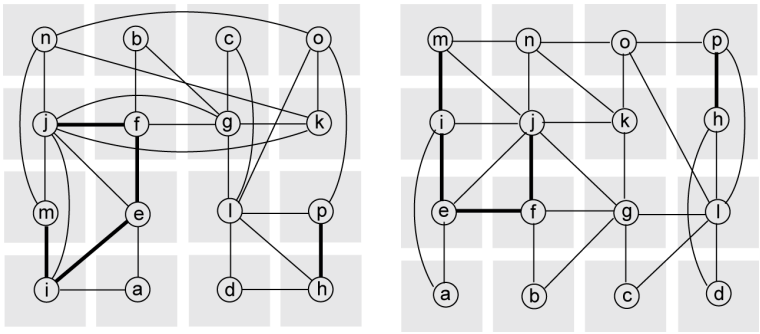
$w(n11) = 1$, $w(n12) = 2$, $w(n13) = 2$. Thus, the total wirelength cost is 23.

Min-Cut Strategies Comparison

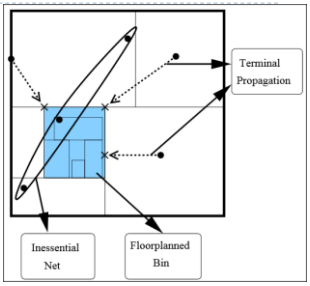
- Quadrature vs recursive bisection + terminal propagation
  - Number of cuts: 6 vs 15
  - Wirelength: 27 vs 23



quadrature bisection

Capo Placer Overview

- Pure recursive bi-partitioning placer
  - Multi-level FM for instances with > 200 cells
  - Flat FM for instances with 35-200 cells
  - Branch-and-bound for instances with < 35 cells
- Careful handling of partitioning tolerance
  - Uncorking: Prevent large cells from blocking smaller cells to move
  - Repartitioning: Several FM calls with decreasing tolerance
  - Block splitting heuristics: Higher tolerance for vertical cut
  - Hierarchical tolerance computation: Instance with more whitespace can have a bigger partitioning tolerance
- Implementation with standard interfaces
  - LEF/DEF and GSRC



Partitioning Tolerance = size ratio

Floorplanning uses Parquet tool – takes place bottom-up, based on failure

Terminal propagation during the floorplanning of a placement bin. The shaded placement bin is being floorplanned because of large/many macros. Dotted lines depict the external connections to objects inside the bin, being propagated as terminals to the placement bin boundaries. An inessential net for floorplanning is also shown. The shown 3-pin inessential net has no consequence on the florplanning decision for HPWL minimization and is removed when forming the floorplanning problem

If the bounding box of any multi-pin net covers the bounding box of the entire bin, then this net will have no consequence on the HPWL minimization during floorplanning. Such a net is treated as inessential and ignored when forming the floorplanning problem.

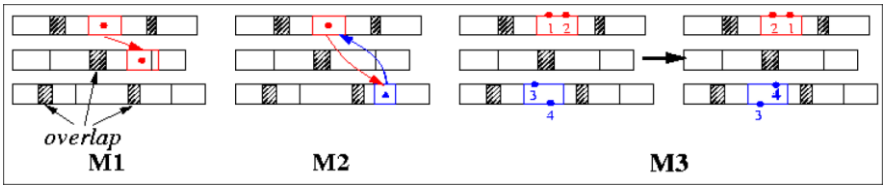
Clustering is based on Hmetis

Placement by Simulated Annealing – TW6

- Sechen and Sangiovanni-Vincentelli, "The TimberWolf placement and routing package," *IEEE J. Solid-State Circuits*, Feb. 1985; "TimberWolf 3.2: A new standard cell placement and global routing package," DAC-86.
- iTools: http://www.twolf.com
- TimberWolf 6.0 – allows overlaps
- TimberWolf: Stage 1
  - Modules are moved between different rows, as well as within the same row
  - Modules overlaps are allowed
  - When the temperature is reached below a certain value, stage 2 begins
- TimberWolf: Stage 2
  - Remove overlaps
  - Annealing process continues, but only interchanges adjacent modules within the same row

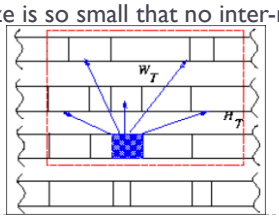
Timberwolf 6.0 Solution Space and Neighborhood Structure

- **Solution Space:** All possible arrangements of the modules into rows, possibly with overlaps.
- **Neighborhood Structure:** 3 types of moves
  - *M*1: Displace a module to a new location.
  - *M*2: Interchange two modules.
  - *M*3: Change the orientation of a module.



Timberwolf 6.0 Solution Space and Neighborhood Structure

- TimberWolf first tries to select a move between *M1* and *M2*: *Prob*(*M*1) = 0.8, *Prob*(*M*2) = 0.2.
- If a move of type *M*1 is chosen and it is rejected, then a move of type *M*3 for the same module will be chosen with probability 0.1
- Restrictions: (1) what row for a module can be displaced? (2) what pairs of modules can be interchanged?
- **Key: Range Limiter**
  - At the beginning, (W_T, H_T) is big enough to contain the whole chip.
  - Window size shrinks as the temperature decreases. Height, width are proportional to *log*(*T*)
  - Stage 2 begins when window size is so small that no inter-row module interchanges are possible



Timberwolf 6.0 Cost Function

- Cost Function C = C1 + C2 + C3

$$C1 = \sum_{i \in Nets} (\alpha_i w_i + \beta_i h_i) \qquad C2 = \gamma \sum_{i \neq j} O_{ij}^2 \qquad C3 = \delta \sum_{r \in rows} |L_r - D_r|$$

- C1: total estimated wirelength
  - α_i, β_i are horizontal and vertical weights respectively
    - For $\alpha_i = 1, \beta_i = 1$ ➔ (1/2) x perimeter of BB of Net I
  - Critical nets: increase α_i, β_i
  - If vertical wirings are "cheaper", use smaller vertical weights
- C2: penalty function for module overlaps
  - where γ is penalty weight, and O_{ij} is the amount of overlaps in x-dimension between modules *i* and *j*
- C3: penalty function controlling row length
  - where δ is the penalty weight, D_r is the desired row length and L_r is the sum of the widths of modules in row r

Timberwolf 6.0 Annealing Schedule

- $T_k = r_k T_{k-1}$
- r_k increases from 0.8 to a maximum value of 0.94, and then decreases to 0.8
- At each T, a total number of $\underline{nP}$ attempts are made
  - Where n is the number of standard-cells
  - P is a user specified constraint
- Termination Condition
  - $T < 0.1$

TimberWolf 7.0

- Cost Function C = C1 + Cs
- C1: total estimated wirelength due to move,
- Cs: additional wirelength due to inter-row cell shifting
  - Number of cells shifted per row are minimized
    - move to the left or to the right accordingly
- Hierarchical placement approach is used
  - Netlist is clustered twice in a recursive fashion
  - The top-level (= second-level) clusters are first placed during the high annealing temperature region
  - These clusters are then decomposed to reveal the first-level clusters
  - The placement among the first-level clusters is refined during the mid annealing temperature region
  - Lastly, the first-level clusters are decomposed to reveal the original gate-level netlist
  - TimberWolf 7.0 refines this gate-level placement during the low annealing temperature region

C1, Cs are really Deltas here = DWL, DWs
Cs is estimated to reduce computational complexity
Shift amount is + for right, - for left

TimberWolf 7.0

- Cost Function C = C1 + Cs
- C1: total estimated wirelength due to move,
- Cs: additional wirelength due to inter-row cell shifting
  - Number of cells shifted per row are minimized
    - move to the left or to the right accordingly
- Cs is estimated: for cell z to be shifted:
  - **Model A:** for each net incident to z, we find two "break points" a and b (-z), which defines range [a, b], where the wirelength of the net
    - (1) does not change if z is located within the range, (2) increases if z is located to the right of b, and (3) decreases if z is located to the left of a;
    - the superposition of break points estimates WL change
  - **Model B:** we first compute the "gradient" of *z* as: $gradient(z) = \sum_{i \in N_z} D_i(0)$
    - where *Nz* denotes the nets incident to *z*, *Di*(0) denotes the "rate of wirelength change of net *i* measured at the origin";
    - If *z* is at the left boundary of the bounding box of net *i*, *Di*(0) = −1, if at the right boundary, *Di*(0) = 1, else *Di*(0) = 0
    - Once the gradient of all cells to be shifted is computed, Cs is estimated as follows:

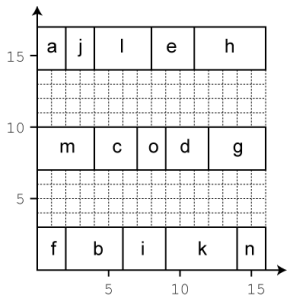
$$C_s = \sum_{j \in shiftedcell} gradient(j) \,.\, shift\_amount(j)$$

C1, Cs are really Deltas here = DWL, DWs
Cs is estimated to reduce computational complexity
Shift amount is + for right, - for left

TimberWolf Placement Example

- Perform TimberWolf placement
  - Based on the given standard cell placement
  - Initial HPBB wirelength = x

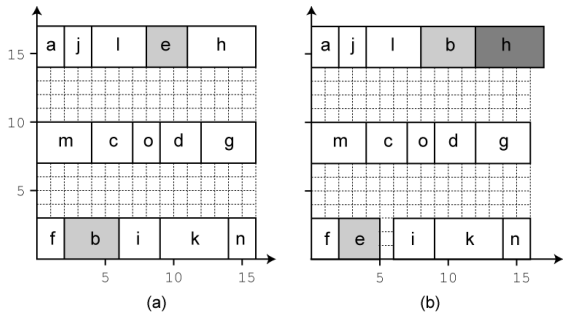
$n_1 = \{a, e, g\}$
$n_2 = \{f, o\}$
$n_3 = \{b, c, k, n\}$
$n_4 = \{d, h, i\}$
$n_5 = \{j, l, m\}$
$n_6 = \{d, k, j\}$
$n_7 = \{c, e, f, h, n\}$
$n_8 = \{d, l\}$
$n_9 = \{b, g, i, m\}$
$n_{10} = \{a, k, o\}$



*n*1 = 12+7 = 19, *n*2 = 7+7 = 14, *n*3 = 12+7 = 19, *n*4 = 5+14 = 19, *n*5 = 4+7 = 11, *n*6 = 7+14 = 21, *n*7 = 14+14 = 28, *n*8 = 5+7 = 12, *n*9 = 12+7 = 19, *n*10 = 9+14 = 23.

TimberWolf Placement Example – Swap (b, e)

- Swap node *b* and e
  - We shift node *h*: on the shorter side of the receiving row
  - Node *b* included in nets $\{n_3, n_9\}$, and e in $\{n_1, n_7\}$



$n_1 = \{a, e, g\}$
$n_2 = \{f, o\}$
$n_3 = \{b, c, k, n\}$
$n_4 = \{d, h, i\}$
$n_5 = \{j, l, m\}$
$n_6 = \{d, k, j\}$
$n_7 = \{c, e, f, h, n\}$
$n_8 = \{d, l\}$
$n_9 = \{b, g, i, m\}$
$n_{10} = \{a, k, o\}$

TimberWolf Placement Example – Swap (b, e)

- ΔW = wirelength change from swap

Let $w(x)$ and $w'(x)$ respectively denote the wirelength before and after the swap. Then,

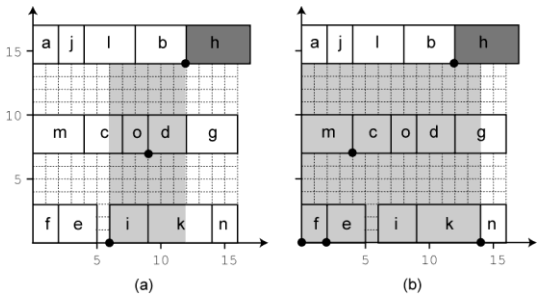
$$\Delta(n_3) = w'(n_3) - w(n_3) = 24 - 19 = 5$$
$$\Delta(n_9) = w'(n_9) - w(n_9) = 26 - 19 = 7$$
$$\Delta(n_1) = w'(n_1) - w(n_1) = 26 - 19 = 7$$
$$\Delta(n_7) = w'(n_7) - w(n_7) = 28 - 28 = 0$$

Thus,

$$\Delta W = \Delta(n_3) + \Delta(n_9) + \Delta(n_1) + \Delta(n_7) = 19$$

TimberWolf Placement Example – Swap (b, e)

- ΔWs = wirelength change from shifting
  - *h* is shifted and included in n_4 = {*d*,*h*,*i*} and n_7 ={*c*,*e*,*f*,*h*,*n*}
  - *h* is on the right boundary of n_4: *gradient*(*h*)++
  - *h* is not on any boundary of n_7: no further change on *gradient*(*h*)



(a) BB of n4 with h on RHS boundary
(b) BB of n7 with h not on any boundary

TimberWolf Placement Example – Swap (b, e)

Thus, $gradient(h) = 1$. Since h is shifted to the right by 1

$$shift\_amount(h) = 1$$

Thus,

$$\Delta W_S = gradient(h) \cdot shift\_amount(h) = 1 \cdot 1 = 1$$

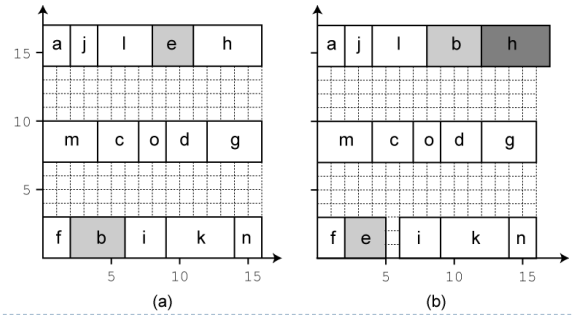
Based on the calculation of ΔW and ΔW_S, we get

$$\Delta C = \Delta W + \Delta W_S = 19 + 1 = 20$$

TimberWolf Placement Example – Swap (b, e)

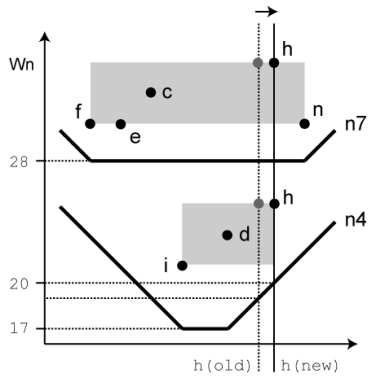
- How accurate is ΔWs estimation?
  - Node *h* is included in n_4 = {*d*,*h*,*i*} and n_7 ={*c*,*e*,*f*,*h*,*n*}
  - Real change is also 1: accurate estimation

$$w'(n_4) - w(n_4) + w'(n_7) - w(n_7) = 20 - 19 + 28 - 28 = 1$$



TimberWolf Placement Example – Swap (b, e) – Model A

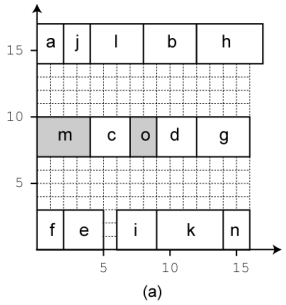
- Based on piecewise linear graph
  - Shifting *h* causes the wirelength of n_4 to increase by 1 (19 to 20) and no change on n_7 (stay at 28)



n7 above, n4 below – break points for n7: f, n, for n4: i, d

TimberWolf Placement Example – Swap (m, n)

- Swap node *m* and *o*
  - We shift node *d* and *g*: on the shorter side of the receiving row
  - Node *m* included in nets $\{n_5, n_9\}$, and *o* in $\{n_2, n_{10}\}$



$$
\begin{aligned}
n_1 &= \{a, e, g\} \\
n_2 &= \{f, o\} \\
n_3 &= \{b, c, k, n\} \\
n_4 &= \{d, h, i\} \\
n_5 &= \{j, l, m\} \\
n_6 &= \{d, k, j\} \\
n_7 &= \{c, e, f, h, n\} \\
n_8 &= \{d, l\} \\
n_9 &= \{b, g, i, m\} \\
n_{10} &= \{a, k, o\}
\end{aligned}
$$

TimberWolf Placement Example – Swap (m, n)

- ΔW = wirelength change from swap

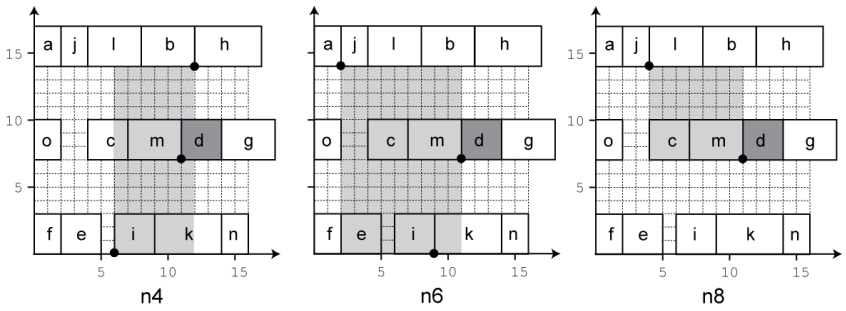
$$\Delta(n_5) = w'(n_5) - w(n_5) = 12 - 11 = 1$$
$$\Delta(n_9) = w'(n_9) - w(n_9) = 22 - 26 = -4$$
$$\Delta(n_2) = w'(n_2) - w(n_2) = 7 - 14 = -7$$
$$\Delta(n_{10}) = w'(n_{10}) - w(n_{10}) = 23 - 23 = 0$$

Thus,

$$\Delta W = \Delta(n_5) + \Delta(n_9) + \Delta(n_2) + \Delta(n_{10}) = -10$$

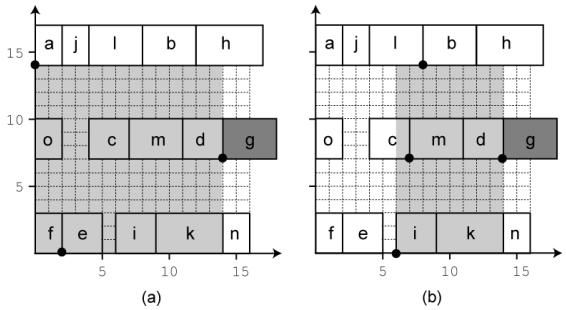
TimberWolf Placement Example – Swap (m, n)

- Cell *d* and *g* are shifted
  - *d* is included in n_4 = {*d*,*h*,*i*}, n_6 ={*d*,*k*,*j*}, and n_8 ={*d*,*l*}
  - *d* is on the right boundary of n_6 and n_8
  - So, *gradient*(*d*) = 2



TimberWolf Placement Example – Swap (m, n)

- Cell *d* and *g* are shifted
  - *g* is included in n_1 = {*a*,*e*,*g*}, and n_9 ={*b*,*g*,*i*,*m*}
  - *g* is on the right boundary of n_1 and n_9
  - So, *gradient*(*g*) = 2



TimberWolf Placement Example – Swap (m, n)

Both cell d and g are shifted to the right by 2. Thus,

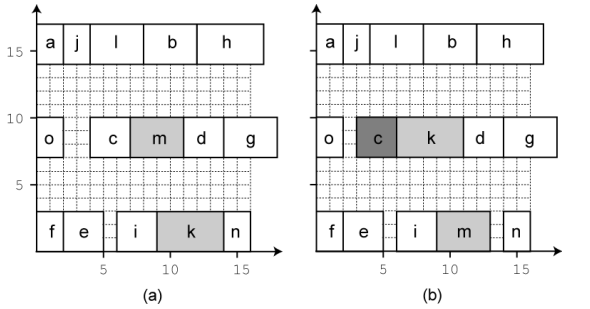
$$\begin{aligned} \Delta W_S &= gradient(d) \cdot shift\_amount(d) + \\ & \quad gradient(g) \cdot shift\_amount(g) = 2 \cdot 2 + 2 \cdot 2 = 8 \end{aligned}$$

Based on the calculation of ΔW and ΔW_S, we get

$$\Delta C = \Delta W + \Delta W_S = -10 + 8 = -2$$

TimberWolf Placement Example – Swap (k, m)

- Swap node *k* and *m*
  - We shift node *c*: on the shorter side of the receiving row
  - Node *k* included in nets $\{n_3, n_6, n_{10}\}$, and *m* in $\{n_5, n_9\}$



$n_1 = \{a, e, g\}$
$n_2 = \{f, o\}$
$n_3 = \{b, c, k, n\}$
$n_4 = \{d, h, i\}$
$n_5 = \{j, l, m\}$
$n_6 = \{d, k, j\}$
$n_7 = \{c, e, f, h, n\}$
$n_8 = \{d, l\}$
$n_9 = \{b, g, i, m\}$
$n_{10} = \{a, k, o\}$

TimberWolf Placement Example – Swap (k, m)

- ΔW = wirelength change from swap

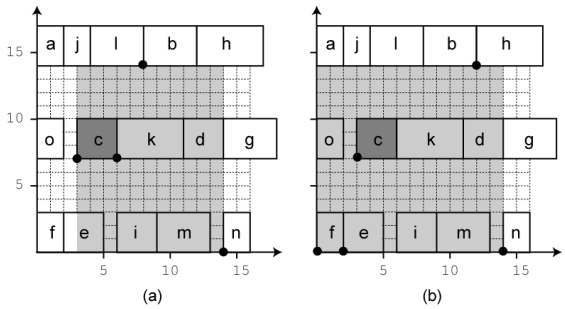
$$\Delta(n_3) = w'(n_3) - w(n_3) = 25 - 24 = 1$$
$$\Delta(n_6) = w'(n_6) - w(n_6) = 16 - 23 = -7$$
$$\Delta(n_{10}) = w'(n_{10}) - w(n_{10}) = 13 - 23 = -10$$
$$\Delta(n_5) = w'(n_5) - w(n_5) = 21 - 12 = 9$$
$$\Delta(n_9) = w'(n_9) - w(n_9) = 22 - 22 = 0$$

Thus,

$$\Delta W = \Delta(n_3) + \Delta(n_6) + \Delta(n_{10}) + \Delta(n_5) + \Delta(n_9) = -7$$

TimberWolf Placement Example – Swap (k, m)

- Cell *c* is shifted
  - *c* is included in n_3 = {*b*,*c*,*k*,*n*} and n_7 ={*c*,*e*,*f*,*h*,*n*}
  - *c* is on the left boundary of n_3
  - So, *gradient*(*c*) = −1



TimberWolf Placement Example – Swap (k, m)

Since c is shifted to the left by 1,

$$shift\_amount(c) = -1$$

Lastly,

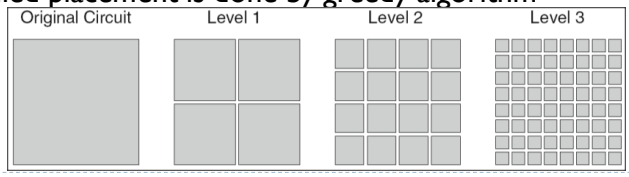
$$\Delta W_S = gradient(c) \cdot shift\_amount(c) = -1 \cdot -1 = 1$$

Based on the calculation of ΔW and ΔW_S, we get

$$\Delta C = \Delta W + \Delta W_S = -7 + 1 = -6$$

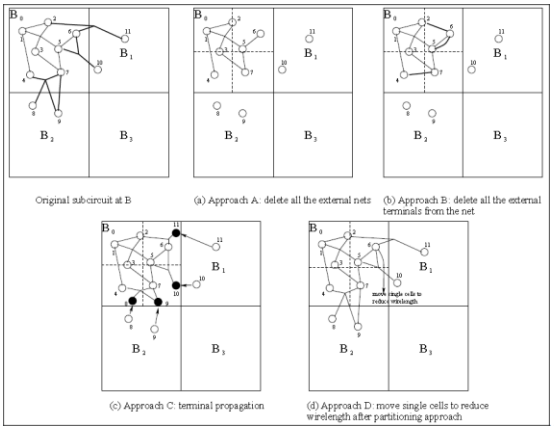
Dragon Placer

- Dragon takes a hybrid approach that combines SA and partitioning
  - Recursive partitioning to reduce the problem size
  - Annealing to refine the solution generated by partitioning
- Top-down hierarchical placement
  - hMetis to recursively quadrisect into 4h bins at level h
  - Swapping of bins at each level by SA to minimize WL
  - Terminates when each bin contains < 7 cells
  - At final stage of GP, switch single cells locally among bins by SA to further minimize WL
- Detailed placement is done by greedy algorithm



Dragon Placer

- GP and DP phases
- GP combines WL with min-cut based on 4 approaches



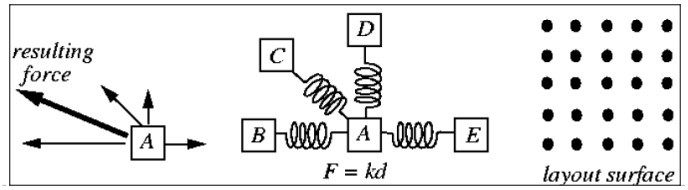
Dragon2000 paper claims approach B is best for the ibm benchmarks

Analytical Placement

- Key Idea: Solve a relaxed placement problem "optimally"
  - Ignore overlaps (fixed later)
  - Adopt linear or non-linear WL estimation
  - Need I/O Pads/Pins to pull cells outwards
- Approaches for overlap removal
  - Cell spreading
  - Legalization
- Pros: very good quality for existing benchmarks
- Cons: hard to handle big macros rotation constraints, region constraints, blockages, etc.

Force-Directed Methods

- Hanan & Kurtzberg, "Placement techniques," in *Design Automation of Digital Systems*, Breuer, Ed, 1972
- Quinn, Jr. & Breuer, "A force directed component placement procedure for printed circuit boards," *IEEE Trans. Circuits and Systems*, June 1979
- Similar to Graph Drawing Algorithms
- Reducing the placement problem to solving a set of simultaneous linear equations to determine equilibrium locations for cells
- Analogy to Hooke's law: $F = kd$, F: force, k: spring constant, d: distance.
- Goal: Map cells to the layout surface.

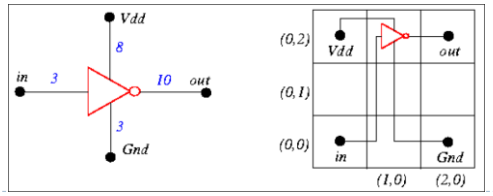


Force-Directed Approaches: Zero-Force Target Location

- Cell *i* connects to several cells *j's* at distances *dij*, by wires of weights *wij's*
  - Total Force: $F_i = \sum_j w_{ij} d_{ij}$
- The zero-Force target location can be determined by equating the x- and y- components of forces to zero:

$$\sum_j w_{ij}(x_j - \hat{x}_i) = 0 \Leftrightarrow \hat{x}_i = \frac{\sum_j w_{ij}\, x_j}{\sum_j w_{ij}}$$

- In the example, $\hat{x}_i = \frac{8x0+10x2+3x0+3x2}{8+10+3+3}$=1.083, $\hat{y}_i$=1.5



Force-Directed Placement

- Approach I (constructive):
  - Start with an initial placement
  - Compute the zero-force locations for all cells.
  - Apply **linear assignment (matching)** to determine the "ideal" locations for the cells
- Approach II (can be constructive or iterative):
  - Start with an initial placement.
  - elect a "most profitable" cell *p* (e.g., maximum *F*, critical cells) and place it in its zero-force location
  - "Fix" placement if the zero-force location has been occupied by another cell *q*.
- Popular options to fix:
  - **Ripple move:** place *p* in the occupied location, compute a new zero-force location for *q*, …
  - **Chain move:** place *p* in the occupied location, move *q* to an adjacent location, …
  - Move *p* to a free location close to *q*.

Linear matching probably means move its cell to its closest zero-force location

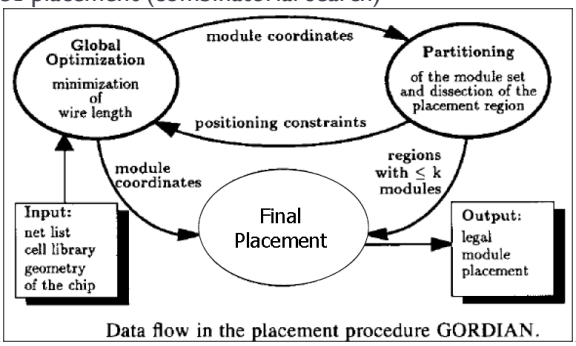
Example of Force-Directed Placement Algorithm

```
Algorithm: Force-Directed_Placement

1 begin
2 Compute the connectivity for each cell;
3 Sort the cells in decreasing order of their connectivities into list L;
4 while (IterationCount < IterationLimit) do
5     Seed ← next module from L;
6     Declare the position of the seed vacant;
7         while (EndRipple = FALSE) do
8             Compute target location of the seed;
9             case the target location
10            VACANT:
11                Move seed to the target location and lock;
12                EndRipple ← TRUE; AbortCount ← 0;
13            SAME AS PRESENT LOCATION:
14                EndRipple ← TRUE; AbortCount ← 0;
15            LOCKED:
16                Move selected cell to the nearest vacant location;
17                EndRipple ← TRUE; AbortCount ← AbortCount + 1;
18                if (AbortCount > AbortLimit) then
19                    Unlock all cell locations;
19                    IterationCount ← IterationCount + 1;
20            OCCUPIED AND NOT LOCKED:
21                Select cell as the target location for next move;
22                Move seed cell to target location and lock the target location;
23                EndRipple ← FALSE; AbortCount ← 0;
26 end
```

Gordian Placement Algorithm

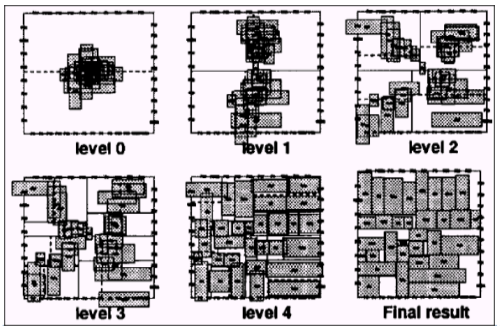
- Kleinhans, Sigl and Johannes, "GORDIAN:VLSI placement by quadratic programming and slicing optimization," TCAD, October 91 (ICCAD-90).
  - Global placement (analytical scheme)
  - Detailed placement (combinatorial search)



Data flow in the placement procedure GORDIAN.

Gordian Global Placement Optimization

- Apply QP (Quadratic Programming)
  - Objective Function: WL (Wire Length)
  - **Squared Distance** between Modules and Top-Level I/O Pins
- Constraints:
  - Centers of the regions of a partitioning level
  - **Cell Spreading Scheme**



QP Objective Function - 1

- General QP - Minimize: $\frac{1}{2}x^T Ax + b^T x + c$
- **This is equivalent to solving the system:** $Ax = b$
- QP can be formulated independently in x, y directions
- For WL minimization:
  - For any connected cells *i*, *j*, let c_{ij} be the connection weight
  - Cost of a 2-point net between cells *i* and *j*:

$$c_{ij}((x_i - x_j)^2 + (y_i - y_j)^2)$$

  - **Ignoring cell overlapping**
- Total Cost:

$$\frac{1}{2}x^T Ax + d_x^T x + \frac{1}{2}y^T Ay + d_y^T + c$$

We may also put ½ in front of the 2-point net cost

QP Objective Function - 1

- For WL minimization: $\frac{1}{2}x^T Ax + b^T x + c$
  - For any connected cells *i*, *j*, let c_{ij} be the connection weight
  - Cost of a 2-point net between cells *i* and *j*:

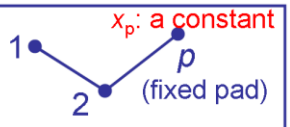
$$c_{ij}((x_i - x_j)^2 + (y_i - y_j)^2$$

- Top-Level I/Os possess **fixed coordinates**
  - **Necessary for Analytical Placement Algorithms**

Horizontal cost

$$= \frac{1}{2}c_{12}(x_1 - x_2)^2 + \frac{1}{2}c_{2p}(x_2 - x_p)^2$$

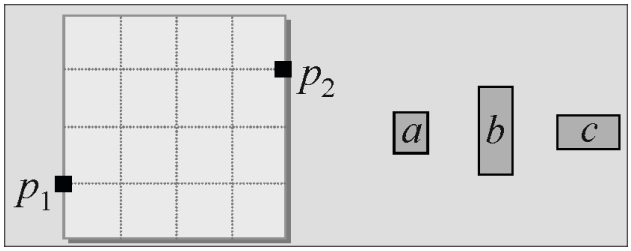
$$= \frac{1}{2}\begin{pmatrix} x_1 & x_2 \end{pmatrix}\begin{pmatrix} c_{12} & -c_{12} \\ -c_{12} & c_{12} + c_{2p} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 & -c_{2p}x_p \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \frac{1}{2}c_{2p}x_p^2$$



We may also put ½ in front of the 2-point net cost

Simple QP Formulation Example

- Given a placement P with two fixed I/O pins
  - p1 @ (100, 175)
  - p2 @ (200, 225), and
- And three standard-cells a, b, c and 4 nets N1-N4:
  - N1(P1, a), N2(a, b), N3(b, c) and N4(c, P2)
- Find the Optimal coordinates of blocks a, b, c



Simple QP Formulation Example

- Formulate Cost in x direction:

$$L_x(P) = (100 - x_a)^2 + (x_a - x_b)^2 + (x_b - x_c)^2 + (x_c - 200)^2$$

$$\frac{\partial L_x(P)}{x_a} = -2(100 - x_a) + 2(x_a - x_b) = 4x_a - 2x_b - 200 = 0$$

$$\frac{\partial L_x(P)}{x_b} = -2(x_a - x_b) + 2(x_b - x_c) = -2x_a + 4x_b - 2x_c = 0$$

$$\frac{\partial L_x(P)}{x_c} = -2(x_b - x_c) + 2(x_c - 200) = -2x_b + 4x_c - 400 = 0$$

- Put it in Matrix Form Ax = b

$$\begin{bmatrix} 4 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} 200 \\ 0 \\ 400 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 200 \end{bmatrix}$$

- Solution: $x_a = 125, x_b = 150, x_c = 175$

Consider p1 -> (a, b), a->c, b->d, c->d, d->p2 with and without clique model

Simple QP Formulation Example

- Formulate Cost in y direction:

$$L_y(P) = (175 - y_a)^2 + (y_a - y_b)^2 + (y_b - y_c)^2 + (y_c - 225)^2$$

$$\frac{\partial L_y(P)}{y_a} = -2(175 - y_a) + 2(y_a - y_b) = 4y_a - 2y_b - 350 = 0$$

$$\frac{\partial L_y(P)}{y_b} = -2(y_a - y_b) + 2(y_b - y_c) = -2y_a + 4y_b - 2y_c = 0$$

$$\frac{\partial L_y(P)}{y_c} = -2(y_b - y_c) + 2(y_c - 225) = -2y_b + 4y_c - 450 = 0$$

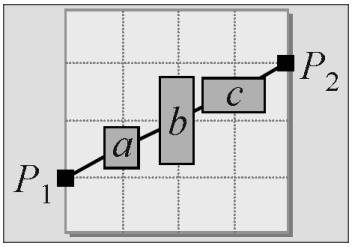
- Put it in Matrix Form Ax = b

$$\begin{bmatrix} 4 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} y_a \\ y_b \\ y_c \end{bmatrix} = \begin{bmatrix} 350 \\ 0 \\ 450 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} y_a \\ y_b \\ y_c \end{bmatrix} = \begin{bmatrix} 175 \\ 0 \\ 225 \end{bmatrix}$$

- Solution: y_a = 187.5, y_b = 200, y_c = 212.5

Simple QP Formulation Example

- Given a placement P with two fixed I/O pins
  - p1 @ (100, 175)
  - p2 @ (200, 225), and
- And three standard-cells a, b, c and 4 nets N1-N4:
  - N1(P1, a), N2(a, b), N3(b, c) and N4(c, P2)
- Find the Optimal coordinates of blocks a, b, c

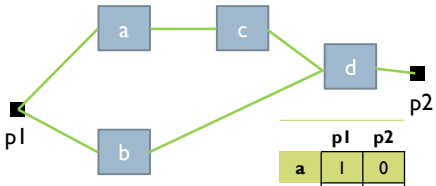


Another Simple QP Formulation Example

- Given a placement P with two fixed I/O pins
  - p1 @ (100, 170)
  - p2 @ (200, 300), and

Point to Point Model

- And four standard-cells a, b, c, d and 5 nets N1-N5:
  - N1(P1, a, b), N2(a, c), N3(b, d), N4(c, d), N5(d, P2)
- Find the Optimal coordinates of blocks a, b, c, d



	p1	p2
a	1	0
b	1	0
c	0	0
d	0	1

	a	b	c	d
a	2	0	-1	0
b	0	2	0	-1
c	-1	0	2	-1
d	0	-1	-1	3

Dx	Dy
100	170
100	170
0	0
200	300

Consider p1 -> (a, b), a->c, b->d, c->d, d->p2 with and without clique model
Nets list (output ->input, i/o -> input/output)
p1: a b
a: c
b: d
c: d
p2: d

Solution for x

http://wims.unice.fr/wims/wims.cgi

	p1	p2
a	1	0
b	1	0
c	0	0
d	0	1

	a	b	c	d
a	2	0	-1	0
b	0	2	0	-1
c	-1	0	2	-1
d	0	-1	-1	3

Dx	Dy
100	170
100	170
0	0
200	300

Linear solver

Hint. Want to enter x^2? Type `x^2` or `x**2`.

You have entered the system

$$\begin{cases} 2x_1 - x_3 = 100 \\ 2x_2 - x_4 = 100 \\ -x_1 + 2x_3 - x_4 = 0 \\ -x_3 - x_2 + 3x_4 = 200 \end{cases}$$

This system has a unique solution, which is

$\{ x_1 = 1300/11, x_2 = 1400/11, x_3 = 1500/11, x_4 = 1700/11 \}$.

- ax = 118, bx = 127, cx = 136, dx = 154

Solution for y

http://wims.unice.fr/wims/wims.cgi

	p1	p2
a	1	0
b	1	0
c	0	0
d	0	1

	a	b	c	d
a	2	0	-1	0
b	0	2	0	-1
c	-1	0	2	-1
d	0	-1	-1	3

Dx	Dy
100	170
100	170
0	0
200	300

Linear solver

Hint. Want to enter x^2? Type `x^2` or `x**2`.

You have entered the system

$$\begin{cases} 2x_1 - x_3 = 170 \\ 2x_2 - x_4 = 170 \\ -x_1 + 2x_3 - x_4 = 0 \\ -x_3 - x_2 + 3x_4 = 300 \end{cases}$$

This system has a unique solution, which is

$\{ x_1 = 2130/11, x_2 = 2260/11, x_3 = 2390/11, x_4 = 2650/11 \}$.

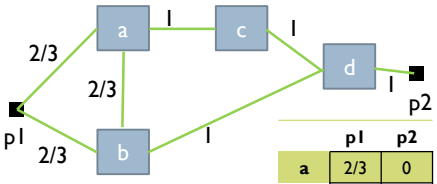
- ay = 193, by = 205, cy = 217, dy = 240

Another Simple QP Formulation Example

- Given a placement P with two fixed I/O pins
  - p1 @ (100, 170)
  - p2 @ (200, 300), and

Clique Model (2/k)

- And four standard-cells a, b, c, d and 5 nets N1-N5:
  - N1(P1, a, b), N2(a, c), N3(b, d), N4(c, d), N5(d, P2)
- Find the Optimal coordinates of blocks a, b, c, d



	p1	p2
a	2/3	0
b	2/3	0
c	0	0
d	0	1

	a	b	c	d
a	7/3	-2/3	-1	0
b	-2/3	7/3	0	-1
c	-1	0	2	-1
d	0	-1	-1	3

Dx	Dy
200/3	340/3
200/3	340/3
0	0
200	300

Consider p1 -> (a, b), a->c, b->d, c->d, d->p2 with and without clique model
Nets list (output ->input, i/o -> input/output)
p1: a b (size = 3) ⇔ **p1->a** (+2/3), **p1 -> b** (+2/3), **a->a** (+2/3), a->b (-2/3), b->a(-2/3), **b->b** (+2/3)
a: c (size = 2)⇔**a->a** (+2/2), a->c (-1), c->a (-1), **c->c** (+1)
b: d (size = 2) ⇔ **b->b** (+1), b->d (-1), d->b (-1), **d->d** (+1)
c: d (size = 2) ⇔ **c->c** (+1), c->d (-1), d->c (-1), **d->d** (+1)
p2: d (size = 2) ⇔ **p2->d** (+1)
Sum of a weights: 2/3 + 2/3 + 1 = 7/3
Sum of b weights: 2/3 + 2/3 + 1 = 7/3
Sum of c weights: 1 + 1 = 2
Sum of d weights: 1 + 1 + 1 = 3

Solution for x

	p1	p2
a	2/3	0
b	2/3	0
c	0	0
d	0	1

	a	b	c	d
a	7/3	-2/3	-1	0
b	-2/3	7/3	0	-1
c	-1	0	2	-1
d	0	-1	-1	3

Dx	Dy
200/3	340/3
200/3	340/3
0	0
200	300

Linear solver

Hint. Want to enter x^2? Type `x^2` or `x**2`.

You have entered the system

$$\begin{cases} 7/3\,x_1 - 2/3\,x_2 - x_3 = 200/3 \\ -2/3\,x_1 + 7/3\,x_2 - x_4 = 200/3 \\ -x_1 + 2\,x_3 - x_4 = 0 \\ -x_2 - x_3 + 3\,x_4 = 200 \end{cases}$$

This system has a unique solution, which is

$\{ x_1 = 5000/39,\ x_2 = 400/3,\ x_3 = 5600/39,\ x_4 = 6200/39 \}$.

- ax = 128, bx = 133, cx = 144, dx = 158

Solution for y

	p1	p2
a	2/3	0
b	2/3	0
c	0	0
d	0	1

	a	b	c	d
a	7/3	-2/3	-1	0
b	-2/3	7/3	0	-1
c	-1	0	2	-1
d	0	-1	-1	3

Dx	Dy
200/3	340/3
200/3	340/3
0	0
200	300

Linear solver

Hint. Want to enter x^2? Type `x^2` or `x**2`.

You have entered the system

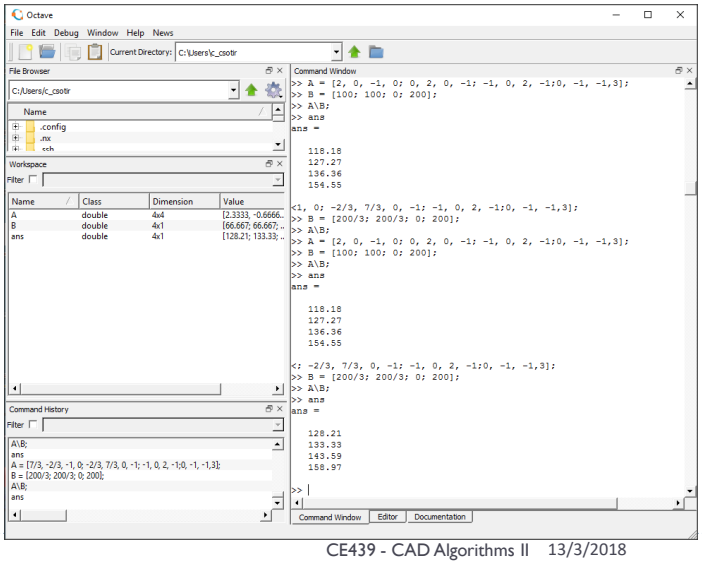
$$\begin{cases} 7/3\,x_1 - 2/3\,x_2 - x_3 = 340/3 \\ -2/3\,x_1 + 7/3\,x_2 - x_4 = 340/3 \\ -x_1 + 2\,x_3 - x_4 = 0 \\ -x_2 - x_3 + 3\,x_4 = 300 \end{cases}$$

This system has a unique solution, which is

$\{ x_1 = 620/3,\ x_2 = 640/3,\ x_3 = 680/3,\ x_4 = 740/3 \}$.

- ay = 206, by = 213, cy = 226, dy = 246

Octave Linear Solver



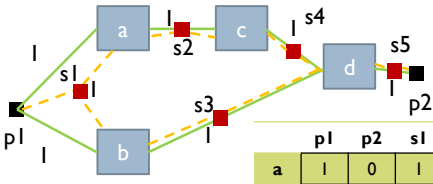
Another Simple QP Formulation Example

- Given a placement P with two fixed I/O pins
  - p1 @ (100, 170)
  - p2 @ (200, 300), and
- And four standard-cells a, b, c, d and 5 nets N1-N5:
  - N1(P1, a, b), N2(a, c), N3(b, d), N4(c, d), N5(d, P2)
- Find the Optimal coordinates of blocks a, b, c, d

Star Model (no weights)

	a	b	c	d
a	4	0	-1	0
b	0	4	0	-1
c	-1	0	4	-1
d	0	-1	-1	6

- s1 @ (120, 210)
- s2 @ (140, 280)
- s3 @ (160, 240)
- s4 @ (180, 260)
- s5 @ (190, 290)



	p1	p2	s1	s2	s3	s4	s5
a	1	0	1	1	0	0	0
b	1	0	1	0	1	0	0
c	0	0	0	1	0	1	0
d	0	1	0	0	1	1	1

Dx	Dy
360	690
380	650
320	540
730	1090

Consider p1 -> (a, b), a->c, b->d, c->d, d->p2 with and without star model
Nets list (output ->input, i/o -> input/output)
Must incorporate Star pins as well, when computing net connectivity:
p1: a b ***s1*** (size = 4)
a: c ***s2*** (size = 3)
b: d ***s3*** (size = 3)
c: d ***s4*** (size = 3)
p2: d ***s5*** (size = 3)
Dx(a) = 100 + 120 + 140 = 360, Dy(a) = 170 + 210 + 280 = 690
Dx(b) = 100 + 120 + 160 = 380, Dy(b) = 200 + 210 + 240 = 650
Dx(c) = 140 + 180 = 320, Dy(c) = 280 + 260 = 540
Dx(d) = 200 + 160 + 180 + 190 =730, Dy(d) = 300 + 240 + 260 + 290 = 1090

Solution for x

	p1	p2	s1	s2	s3	s4	s5
a	1	0	1	1	0	0	0
b	1	0	1	0	1	0	0
c	0	0	0	1	0	1	0
d	0	1	0	0	1	1	1

	a	b	c	d
a	4	0	-1	0
b	0	4	0	-1
c	-1	0	4	-1
d	0	-1	-1	6

Dx	Dy
360	690
380	650
320	540
730	1090

Linear solver

Hint. Want to enter x^2? Type `x^2` or `x**2`.

You have entered the system

$$\begin{cases} 4x_1 - x_3 = 360 \\ 4x_2 - x_4 = 380 \\ -x_1 + 4x_3 - x_4 = 320 \\ -x_3 - x_2 + 6x_4 = 730 \end{cases}$$

This system has a unique solution, which is

$\{ x_1 = 42340/329,\ x_2 = 45270/329,\ x_3 = 50920/329,\ x_4 = 56060/329 \}$.

- ax = 128.7, bx = 137.6, cx = 154.7, dx = 170.4

Solution for y

	p1	p2	s1	s2	s3	s4	s5
a	1	0	1	1	0	0	0
b	1	0	1	0	1	0	0
c	0	0	0	1	0	1	0
d	0	1	0	0	1	1	1

	a	b	c	d
a	4	0	-1	0
b	0	4	0	-1
c	-1	0	4	-1
d	0	-1	-1	6

Dx	Dy
360	690
380	650
320	540
730	1090

Linear solver

Hint. Want to enter x^2? Type `x^2` or `x**2`.

You have entered the system

$$\begin{cases} 4x_1 - x_3 = 690 \\ 4x_2 - x_4 = 650 \\ -x_1 + 4x_3 - x_4 = 540 \\ -x_3 - x_2 + 6x_4 = 1090 \end{cases}$$

This system has a unique solution, which is

$\{ x_1 = 78150/329,\ x_2 = 75100/329,\ x_3 = 85590/329,\ x_4 = 86550/329 \}$.

- ay = 237, by = 228.27, cy = 260.1, dy = 263.1

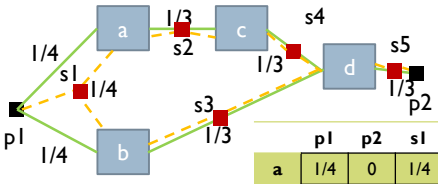
Another Simple QP Formulation Example

- Given a placement P with two fixed I/O pins
  - p1 @ (100, 170)
  - p2 @ (200, 300), and
- And four standard-cells a, b, c, d and 5 nets N1-N5:
  - N1(P1, a, b), N2(a, c), N3(b, d), N4(c, d), N5(d, P2)
- Find the Optimal coordinates of blocks a, b, c, d

Star Model (1/k)

	a	b	c	d
a	7/6	0	-1/3	0
b	0	7/6	0	-1/3
c	-1/3	0	4/3	-1/3
d	0	-1/3	-1/3	2

- s1 @ (120, 210)
- s2 @ (140, 280)
- s3 @ (160, 240)
- s4 @ (180, 260)
- s5 @ (190, 290)



	p1	p2	s1	s2	s3	s4	s5
a	1/4	0	1/4	1/3	0	0	0
b	1/4	0	1/4	0	1/3	0	0
c	0	0	0	1/3	0	1/3	0
d	0	1/3	0	0	1/3	1/3	1/3

Dx	Dy
305/3	2350/12
325/3	365/2
320/3	540/3
730/3	1090/3

Consider p1 -> (a, b), a->c, b->d, c->d, d->p2 with and without star model
Nets list (output ->input, i/o -> input/output)
Must incorporate Star pins as well, when computing net connectivity:
p1: a b ***s1*** (size = 4), p1->a = 1/4, p1->b = 1/4, p1->s1 = 1/4, **s1->a** = +1/4, **s1->b** = +1/4
a: c ***s2*** (size = 3), a->a = +1/3, a->c = -1/3, c->a = -1/3, c->c = +1/3, **s2->a** = +1/3, **s2->c** = +1/3
b: d ***s3*** (size = 3), b->b = +1/3, b->d = -1/3, d->b = -1/3, d->d = +1/3, **s3->b** = +1/3, **s3->d** = +1/3
c: d ***s4*** (size = 3), c->c = +1/3, c->d = -1/3, d->c = -1/3, d->d = +1/3, **s4->c** = +1/3, **s4->d** = +1/3
p2: d ***s5*** (size = 3), p2->d = 1/3, p2->s5 = 1/3, **s5->d** = 1/3
Sum of a weights = 1/4 + 1/4 + 1/3 + 1/3 = 7/6
Sum of b weights = 1/4 + 1/4 + 1/3 + 1/3 = 7/6
Sum of c weights = 1/3 + 1/3 + 1/3 + 1/3 = 4/3
Sum of d weights = 1/3 + 1/3 + 1/3 + 1/3 + 1/3 + 1/3 = 6/3 = 2
Dx(a) = 100 * 1/4 + 120 * 1/4 + 140 * 1/3 = 305/3, Dy(a) = 200 * 1/4 + 210 * 1/4 + 280 * 1/3 = 2350/12
Dx(b) = 100 * 1/4 + 120 * 1/4 + 160 * 1/3 = 325/3, Dy(b) = 200 * 1/4 + 210 * 1/4 + 240 * 1/3 = 365/2
Dx(c) = 140 * 1/3 + 180 * 1/3 = 320/3, Dy(c) = 280 * 1/3 + 260 * 1/3 = 540/3

Dx(d) = 200 * 1/3 + 160 * 1/3 + 180 * 1/3 + 190 * 1/3 =730/3, Dy(d) = 300 * 1/3 + 240 * 1/3 + 260 * 1/3 + 290 * 1/3 = 1090/3

Solution for x

	p1	p2	s1	s2	s3	s4	s5
a	1/4	0	1/4	1/3	0	0	0
b	1/4	0	1/4	0	1/3	0	0
c	0	0	0	1/3	0	1/3	0
d	0	1/3	0	0	1/3	1/3	1/3

	a	b	c	d
a	7/6	0	-1/3	0
b	0	7/6	0	-1/3
c	-1/3	0	4/3	-1/3
d	0	-1/3	-1/3	2

Dx	Dy
305/3	2350/12
325/3	365/2
320/3	540/3
730/3	1090/3

Linear solver

Hint. Want to enter x^2? Type `x^2` or `x**2`.

You have entered the system

$$\begin{cases} 7/6\ x_1 - 1/3\ x_3 = 305/3 \\ 7/6\ x_2 - 1/3\ x_4 = 325/3 \\ -1/3\ x_1 + 4/3\ x_3 - 1/3\ x_4 = 320/3 \\ -1/3\ x_3 - 1/3\ x_2 + 2\ x_4 = 730/3 \end{cases}$$

This system has a unique solution, which is

$\{ x_1 = 130450/991, x_2 = 140510/991, x_3 = 154320/991, x_4 = 169710/991 \}$.

- ax = 131.6, bx = 141.7, cx = 155.7, dx = 171.2

Solution for y

	p1	p2	s1	s2	s3	s4	s5
a	1/4	0	1/4	1/3	0	0	0
b	1/4	0	1/4	0	1/3	0	0
c	0	0	0	1/3	0	1/3	0
d	0	1/3	0	0	1/3	1/3	1/3

	a	b	c	d
a	7/6	0	-1/3	0
b	0	7/6	0	-1/3
c	-1/3	0	4/3	-1/3
d	0	-1/3	-1/3	2

Dx	Dy
305/3	2350/12
325/3	365/2
320/3	540/3
730/3	1090/3

Linear solver

Hint. Want to enter x^2? Type `x^2` or `x**2`.

You have entered the system

$$\begin{cases} 7/6\,x_1 - 1/3\,x_3 = 1175/6 \\ 7/6\,x_2 - 1/3\,x_4 = 365/2 \\ -1/3\,x_1 + 4/3\,x_3 - 1/3\,x_4 = 180 \\ -1/3\,x_3 - 1/3\,x_2 + 2\,x_4 = 1090/3 \end{cases}$$

This system has a unique solution, which is

$\{ x_1 = 240425/991,\ x_2 = 229745/991,\ x_3 = 259275/991,\ x_4 = 261535/991 \}$.

- ay = 242.6, by = 231.83, cy = 261.6, dy = 263.9

Net Models Comparison

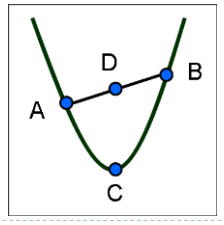
Model	X Coordinate	Y Coordinate
P2P (no weights)	ax = 118, bx = 127, cx = 136, dx = 154	ay = 193, by = 205, cy = 217, dy = 240
CLIQUE (2/n)	ax = 128, bx = 133, cx = 144, dx = 158	ay = 206, by = 213, cy = 226, dy = 246
STAR (no weights)	ax = 128.7, bx = 137.6, cx = 154.7, dx = 170.4	ay = 237, by = 228.27, cy = 260.1, dy = 263.1
STAR (1/n)	ax = 131.6, bx = 141.7, cx = 155.7, dx = 171.2	ay = 242.6, by = 231.83, cy = 261.6, dy = 263.9

Assumes Existing Placement

Convexity

- The QP objective function is a **convex** function
  - If x is a single variable f''(x) ≥ 0
  - If x = [x1 x2 … xn]^T, $x^T H_f x \geq 0$

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

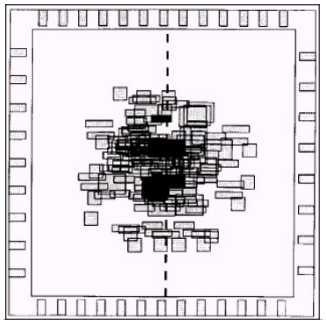


A: $(x, f(x))$
B: $(x', f(x'))$
C: $(cx + (1-c)\,x', f(cx + (1-c)\,x'))$
D: $(cx + (1-c)\,x', cf(x) + (1-c)\,f(x'))$
Convex: $cf(x) + (1-c)\,f'(x') \geq f(cx + (1-c)\,x')$

H is the Hessian Matrix, and is a square matrix of the partial derivatives of f

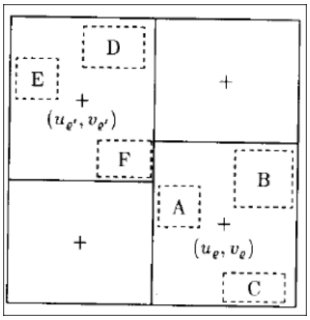
Gordian Partitioning Scheme

- After global optimization, modules are divided into two parts such that the module areas of both subsets are about the same
  - Sort the x- (y-) coordinates of the modules and cut the region vertically (horizontally).



Gordian Partitioning Constraints

- Constraints: make the center of module gravity correspond to the center of the region, $A^{(l)}x = u^{(l)}$
  - each entry in A gives the area ratio of the module in the corresponding region
  - f_m: area of module m, u_r = center of a region r, x_m = module coordinate



$$\mathbf{A}^{(l)} = \begin{array}{c} \\ \varrho \\ \varrho' \\ \\ \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F & \cdots \end{array} \\ \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & * & * & * & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \end{array}$$

$$x_a f_a + x_b f_b + x_c f_c = u_r(f_a + f_b + f_c)$$
$$x_d f_d + x_e f_e + x_f f_f = u_r'(f_d + f_e + f_f)$$

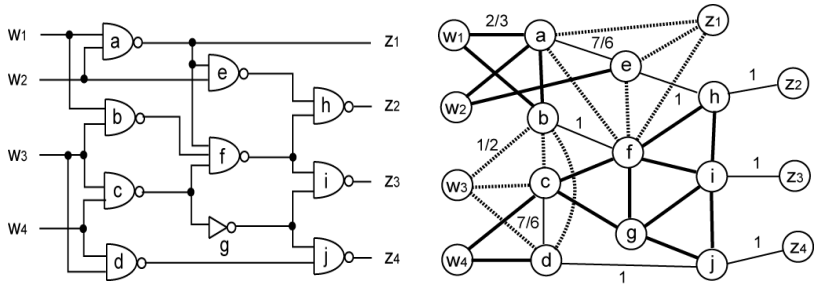
Gordian QP Formulation

- The objective function is a convex quadratic function.
  - C is positive definite.
- Constraints are linear (and thus convex)
- QP has a unique global minimum x*
- QP can be solved optimally in polynomial time
- Ignore the non-overlapping constraint at the global placement stage
- The problem in x- and y-directions can be separated and solved independently

$$QP: \min_x \left(F(x) = \frac{1}{2} x^T A x + d_x^T x;\ A^{(l)} x = u^{(l)} \right)$$

Gordian Placement Example

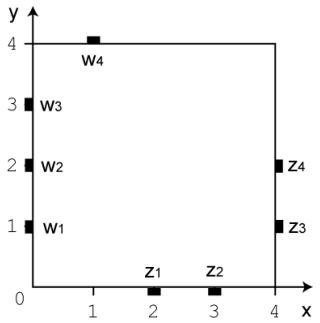
- Perform GORDIAN placement
  - Uniform area and net weight, area balance factor = 0.5
  - Undirected graph model: each edge in *k*-clique gets weight 2/*k*



Thick edges have a weight of 2/3, dotted of ½
Gate a is connected to a 6 clique
Gate a sum of net weights (2/3 + 2/3 + 2/3 + 7/6 +1/2 + 1/2) = (19/6 + 1) = 25/6
Gate e is a member of two nets: (a, e, f, z1), (w2, a, e) = 2/4 + 2/3 = 7/6
It is important to note that the 7/6 factor attempts to account for the two different net connections to e's inputs, one from the w2 input net, the other from the a gate output
Gates with multiple outputs? We discuss this issue later!

Top-Level I/O Placement

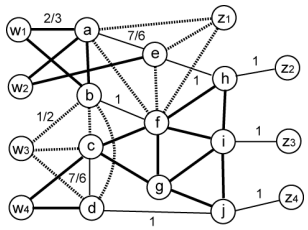
- Necessary for any Analytical Method to work



Matrix A Build-up – **Adjacency Matrix**

- Shows connections among movable nodes
  - Among nodes *a* to *j*

$$\begin{pmatrix}
0 & \frac{2}{3} & 0 & 0 & \frac{7}{6} & \frac{1}{2} & 0 & 0 & 0 & 0 \\
\frac{2}{3} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & \frac{7}{6} & 0 & \frac{2}{3} & \frac{2}{3} & 0 & 0 & 0 \\
0 & \frac{1}{2} & \frac{7}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\frac{7}{6} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 1 & 0 & 0 \\
\frac{1}{2} & 1 & \frac{2}{3} & 0 & \frac{1}{2} & 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 \\
0 & 0 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & \frac{2}{3} \\
0 & 0 & 0 & 0 & 1 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 & \frac{2}{3} \\
0 & 0 & 0 & 1 & 0 & 0 & \frac{2}{3} & 0 & \frac{2}{3} & 0
\end{pmatrix}$$



The Adjacency Matrix is created based on the I/O, output to input and input to output nets.
Gate connections are counted twice, *e.g.* a->e (output->input), and e->a (input to output).

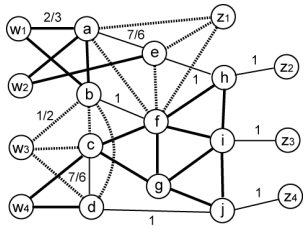
Similar to Graph adjacency matrix, or Component CCs for both input and output connections - **in terms of # of connections**!
To apply net weight, the output->input connections of gate i must be explored and entries (i, j1), (i, j2), … (i, jn) be stored
by symmetry matrix entries (jn, i) … (j2, i), (j1, i) may be copied over

Gates with multiple outputs? We discuss this issue later!

Matrix A Build-up – **Pin Connection Matrix**

- Shows connections between movable nodes and IO
  - Rows = movable nodes, columns = IO (fixed)

$$\begin{pmatrix} \frac{2}{3} & \frac{2}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

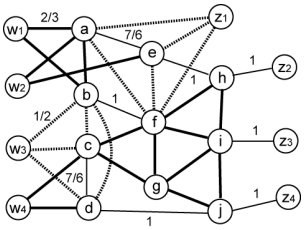


Columns are: w1 w2 w3 …

Matrix A Build-up – **Degree Matrix**

- Based on **both adjacency and pin connection matrices**
  - Sum of entries in the same row (= node degree)

$$\begin{pmatrix} \cdot & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cdot & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cdot & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdot & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdot & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdot & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdot & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot \end{pmatrix}$$



Forming the weighted diagonal requires adding each nodes weighted connections, *e.g.* for node e we must add a->e to both a and e's diagonal entries;
Each net contributes to all its constituent node diagonals!

Clique diagonal calculation: for each net, each pair is connected by net weight. For node a:
(w1, a, b), (w2, a, e), (a, e, f, z1) => 2/3 (w1->a) + 2/3 (a->b) + 2/3 (w2->a) + 2/3 (a->e) + 2/4 (a->e) + 2/4 (a->f) + 2/4 (a->z1) =
8/3 + 3/2 = 16/6 + 9/6 = 25/6

Matrix A Build-up – **Laplacian Matrix**

- L = D – A = Degree Matrix – Adjacency Matrix

$$\begin{pmatrix} \frac{25}{6} & -\frac{2}{3} & 0 & 0 & -\frac{7}{6} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ -\frac{2}{3} & \frac{29}{6} & -\frac{1}{2} & -\frac{1}{2} & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{25}{6} & -\frac{7}{6} & 0 & -\frac{2}{3} & -\frac{2}{3} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{7}{6} & \frac{29}{6} & 0 & 0 & 0 & 0 & 0 & -1 \\ -\frac{7}{6} & 0 & 0 & 0 & \frac{23}{6} & -\frac{1}{2} & 0 & -1 & 0 & 0 \\ -\frac{1}{2} & -1 & -\frac{2}{3} & 0 & -\frac{1}{2} & \frac{31}{6} & -\frac{2}{3} & -\frac{2}{3} & -\frac{2}{3} & 0 \\ 0 & 0 & -\frac{2}{3} & 0 & 0 & -\frac{2}{3} & \frac{8}{3} & 0 & -\frac{2}{3} & -\frac{2}{3} \\ 0 & 0 & 0 & 0 & -1 & -\frac{2}{3} & 0 & \frac{10}{3} & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{3} & -\frac{2}{3} & -\frac{2}{3} & \frac{11}{3} & -\frac{2}{3} \\ 0 & 0 & 0 & -1 & 0 & 0 & -\frac{2}{3} & 0 & -\frac{2}{3} & \frac{10}{3} \end{pmatrix}$$

L, the Laplacian Matrix is defined as: L = D – A, where D is the degree matrix, and A is the adjacency matrix of the graph

Fixed Pin Vectors Build-up

- Based on pin connection matrix and IO location

Each entry i in d_x, denoted $d_{x,i}$, is computed as follows:

$$d_{x,i} = -\sum_j p_{ij} \cdot x(p_j)$$

where p_{ij} denotes the entry of the pin connection matrix, and $x(p_j)$ is the x-coordinate of the corresponding IO pin j.

- Y-direction is defined similarly

The minus is because the pin connection matrix entry p_{ij} is negative
The d vector must be positive

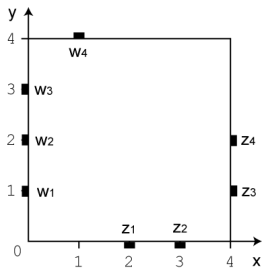
Fixed Pin Vectors Build-up

$$d_{x,1} = -(\frac{2}{3}\cdot 0 + \frac{2}{3}\cdot 0 + 0\cdot 0 + 0\cdot 1 + \frac{1}{2}\cdot 2 + 0\cdot 3 + 0\cdot 4 + 0\cdot 4) = -1$$

By examining the remaining 9 movable cells, we get

$$d_x^T = \begin{pmatrix} -1 & 0 & -\frac{2}{3} & -\frac{2}{3} & -1 & -1 & 0 & -3 & -4 & -4 \end{pmatrix}$$

$$\begin{pmatrix} \frac{2}{3} & \frac{2}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



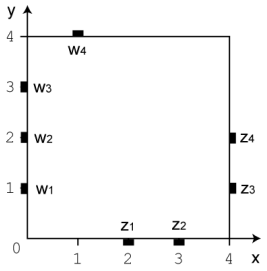
Fixed Pin Vectors Build-up

$$d_{y,1} = -(\frac{2}{3}\cdot 1 + \frac{2}{3}\cdot 2 + 0\cdot 3 + 0\cdot 4 + \frac{1}{2}\cdot 0 + 0\cdot 0 + 0\cdot 1 + 0\cdot 2) = -2$$

By examining the remaining 9 movable cells, we get

$$d_y^T = \begin{pmatrix} -2 & -\frac{13}{6} & -\frac{25}{6} & -\frac{25}{6} & -\frac{4}{3} & 0 & 0 & 0 & -1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} \frac{2}{3} & \frac{2}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



Gordian Level 0 QP Formulation

- No constraints necessary

Minimize

$$\phi(x) = \frac{1}{2}x^T Cx + {d_x}^T x$$

and

$$\phi(y) = \frac{1}{2}y^T Cy + {d_y}^T y$$

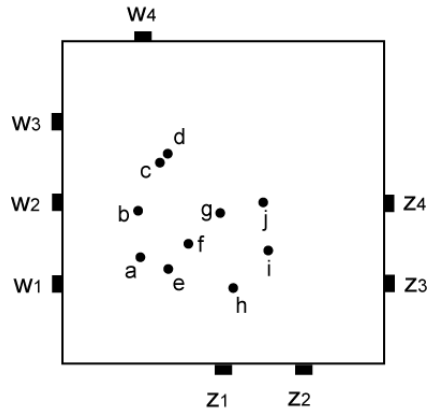
We use MOSEK and obtain the following solution:

$$x^T = \begin{pmatrix} 0.95 & 0.92 & 1.21 & 1.32 & 1.32 & 1.61 & 1.98 & 2.13 & 2.59 & 2.51 \end{pmatrix}$$
$$y^T = \begin{pmatrix} 1.27 & 1.83 & 2.48 & 2.61 & 1.16 & 1.45 & 1.84 & 0.92 & 1.41 & 2.03 \end{pmatrix}$$

- MOSEK Alternatives:
  - **GSL, Intel's MKL**

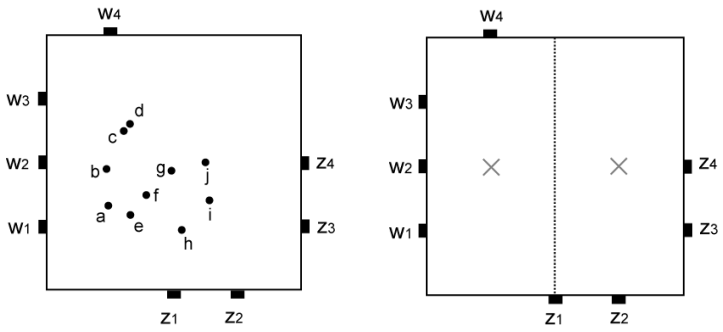
Gordian Level 0 Placement

- Cells with Actual Dimensions will Overlap!



Gordian Level 1 Partitioning

- Perform level 1 partitioning
  - Obtain center locations for center-of-gravity constraints



- $u_x^{(1)} = [1\ 3]^T, u_y^{(1)} = [2\ 2]^T$

Gordian Level 1 Constraint

We first sort the nodes based on their x-coordinates:

$$\{b, a, c, e, d, f, g, h, j, i\}$$

We perform partitioning under $\alpha = 0.5$:

$$S_{\rho'} = \{b, a, c, e, d\},\ S_{\rho''} = \{f, g, h, j, i\}$$

The center location vectors are:

$$u_x^{(1)} = \begin{pmatrix} 1 \\ 3 \end{pmatrix},\ u_y^{(1)} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

We build the matrix $A^{(1)}$ for the center-of-gravity constraint at level $l = 1$:

$$A^{(1)} = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$$

Alpha is the area ratio constraint - check

Gordian Level 1 LQP Formulation

We now solve the following Linearly constrained QP (LQP) to obtain the new placement for the movable nodes:

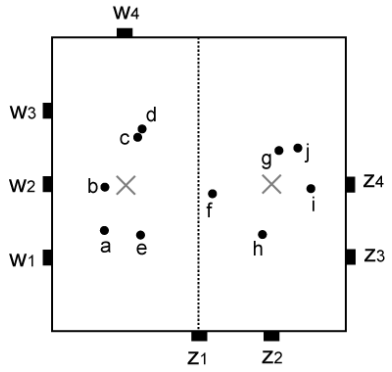
$$\text{Minimize } \phi(x) = \frac{1}{2}x^T Cx + {d_x}^T x, \text{ subject to } A^{(1)} \cdot x = u_x^{(1)}$$

$$\text{Minimize } \phi(y) = \frac{1}{2}y^T Cy + {d_y}^T y, \text{ subject to } A^{(1)} \cdot y = u_y^{(1)}$$

The solutions are as follows:

$$x^T = \begin{pmatrix} 0.70 & 0.71 & 1.17 & 1.21 & 1.22 & 2.17 & 3.10 & 2.84 & 3.56 & 3.33 \end{pmatrix}$$
$$y^T = \begin{pmatrix} 1.34 & 1.94 & 2.66 & 2.76 & 1.30 & 1.83 & 2.45 & 1.32 & 1.91 & 2.49 \end{pmatrix}$$

Gordian Level 1 Placement



Gordian Verification

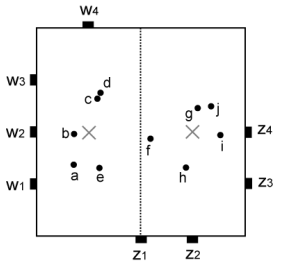
- Verify that the LQP constraints are satisfied in the LHS partition

The following cells are partitioned to the left: $a(0.70, 1.34)$, $b(0.71, 1.94)$, $c(1.17, 2.66)$, $d(1.21, 2.76)$, and $e(1.22, 1.30)$. Thus, the center of gravity is located at:

$$\frac{0.70 + 0.71 + 1.17 + 1.21 + 1.22}{5} = 1.00$$

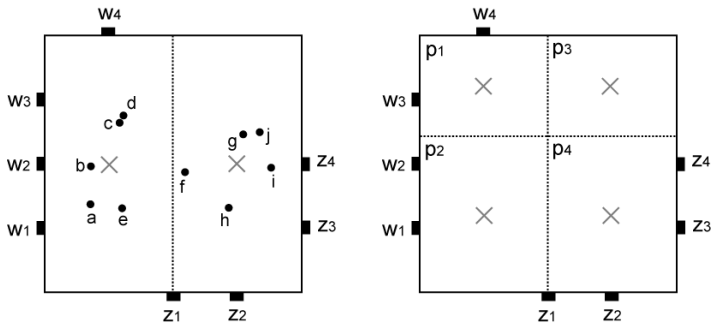
$$\frac{1.34 + 1.94 + 2.66 + 2.76 + 1.30}{5} = 2.00$$

This agrees with the center location $(1, 2)$.



Gordian Level 2 Partitioning

- Add two more cut-lines
  - This results in p_1={c,d}, p_2={a,b,e}, p_3={g,j}, p_4={f,h,i}



Gordian Level 2 Constraint

The center location vectors are:

$$u_x^{(2)} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 3 \end{pmatrix}, \; u_y^{(2)} = \begin{pmatrix} 3.2 \\ 1.2 \\ 3.2 \\ 1.2 \end{pmatrix}$$

Next, we build the matrix $A^{(2)}$ for the center-of-gravity constraint at level $l = 2$. Recall that $p_1 = \{c, d\}$, $p_2 = \{a, b, e\}$, $p_3 = \{g, j\}$, $p_4 = \{f, h, i\}$. Thus,

$$A^{(2)} = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

where the rows denote the partitions p_1 through p_4, and the columns denote the cells a through j.

Gordian Level 2 LQP Formulation

We now solve the following LQP to obtain the placement of the movable nodes:

$$\text{Minimize } \phi(x) = \frac{1}{2}x^T Cx + {d_x}^T x, \text{ subject to } A^{(2)} \cdot x = u_x^{(2)}$$

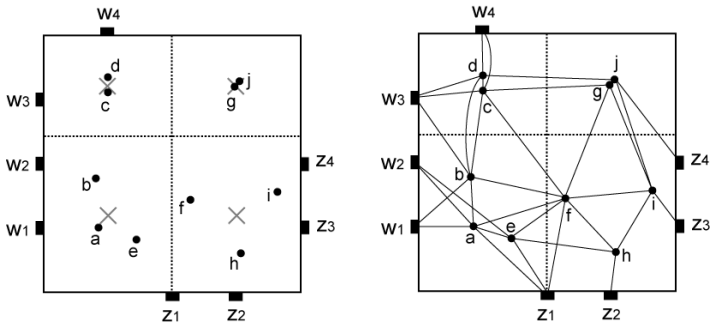
$$\text{Minimize } \phi(y) = \frac{1}{2}y^T Cy + {d_y}^T y, \text{ subject to } A^{(2)} \cdot y = u_y^{(2)}$$

The solutions are as follows:

$$x^T = \begin{pmatrix} 0.83 & 0.78 & 1.00 & 1.00 & 1.39 & 2.28 & 2.89 & 3.06 & 3.66 & 3.11 \end{pmatrix}$$
$$y^T = \begin{pmatrix} 1.01 & 1.78 & 3.08 & 3.32 & 0.82 & 1.44 & 3.18 & 0.59 & 1.57 & 3.22 \end{pmatrix}$$

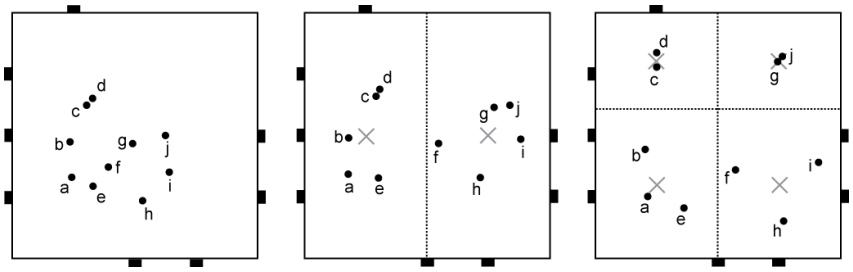
Gordian Level 2 Placement

- Clique-based wiring is shown



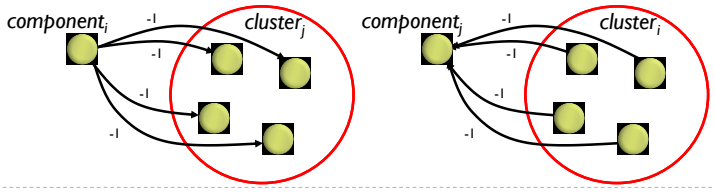
Gordian Example Summary

- Center-of-gravity constraint
  - Helps spread the cells evenly while monitoring WL
  - Removes overlaps among the cells
    - Using actual cell dimension



Clustered Placement QP Formulation

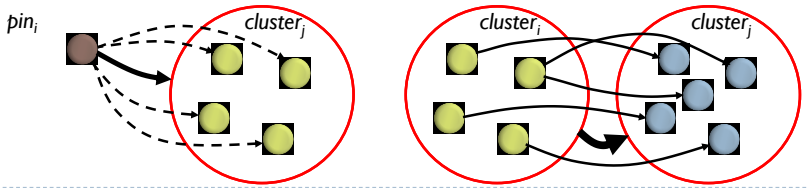
- each cluster is composed of arbitrary set of components
- N Components, M Clusters, T I/Os
  - A Matrix Size is (M + N) x (M + N)
  - Pin Connection Matrix is (M x N) x T
- each (Component → Cluster) Connection gets -1, or -weight in ($component_i$, $cluster_j$) cell of Laplacian Matrix
- each (Cluster → Component) Connection gets -1, or -weight in ($cluster_i$, $component_j$) cell of Laplacian Matrix
- Thus, diagonal A entries (i, i) should **sum the total of relevant** (Component → Cluster) or (Cluster → Component) connections
  - *Magnitude of diagonal entries (i, i), per row, must be > sum of (i, j) connections*



Clustered Placement QP Formulation

- each pin may be connected to multiple cluster components
- multiple (Pin → Cluster), (Cluster → Pin) connections are collapsed to a single (Pin → Cluster), (Cluster → Pin) connections
  - each cluster is represented by a single pin connection column
- Cluster cells may be connected to multiple cells of another cluster
- multiple (Cluster → Cluster) connections must also be collapsed to single (i, j) connections in Laplacian Matrix
  - -1 or –weight
- Thus, diagonal A entries (i, i) should **consider collapsed** (Cluster → Cluster) connections
  - *Magnitude of diagonal entries (i, i), per row, must be > sum of (i, j) connections*

inconsistent!



The issue with this is that it creates an asymmetrical Laplacian Matrix representation, with component to multiple clustered components, and vice versa, being assigned very large weights in their respective Degree Matrix (diagonal) entry, as well as large edge weights in their Adjacency Matrix entries.

For example, in the counter-MAPPED7.v example, the Adjacency Matrix entries for the connection between counter/U4, a reset buffer, and Cluster 0, (3, 9), and (9, 3) are -8, and the Degree Matrix entry for Cluster 0, (9, 9) is 26.

To correct this asymmetry it is necessary to also collapse component to cluster and cluster to component multiple connections to single ones.

Clustered Placement QP Formulation

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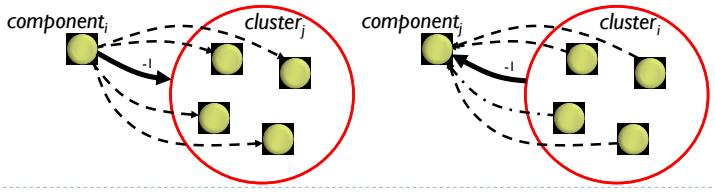

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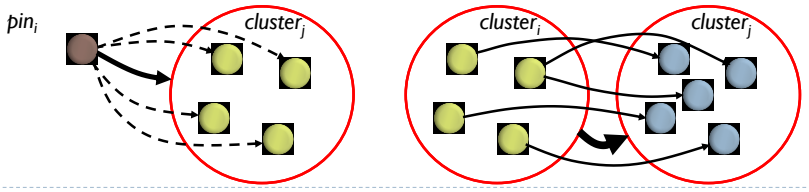
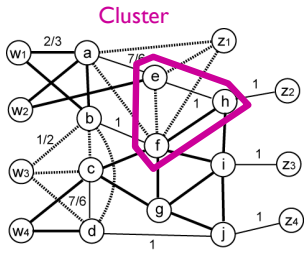


Matrix A Build-up – **Adjacency Matrix**

- Shows connections among movable nodes
  - Among nodes *a* to *j*

$$\begin{pmatrix}
0 & \frac{2}{3} & 0 & 0 & \frac{7}{6} & \frac{1}{2} & 0 & 0 & 0 & 0 \\
\frac{2}{3} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & \frac{7}{6} & 0 & \frac{2}{3} & \frac{2}{3} & 0 & 0 & 0 \\
0 & \frac{1}{2} & \frac{7}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\frac{7}{6} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 1 & 0 & 0 \\
\frac{1}{2} & 1 & \frac{2}{3} & 0 & \frac{1}{2} & 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 \\
0 & 0 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & \frac{2}{3} \\
0 & 0 & 0 & 0 & 1 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 & \frac{2}{3} \\
0 & 0 & 0 & 1 & 0 & 0 & \frac{2}{3} & 0 & \frac{2}{3} & 0
\end{pmatrix}$$



The Adjacency Matrix is created based on the I/O, output to input and input to output nets.
Gate connections are counted twice, *e.g.* a->e (output->input), and e->a (input to output).

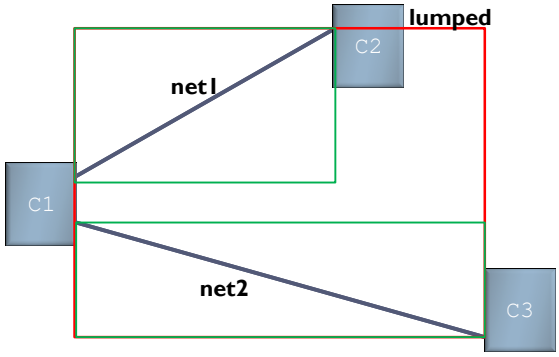
Similar to Graph adjacency matrix, or Component CCs for both input and output connections

Gates with multiple outputs? We discuss this issue later!

For the purple cluster, what are the changes to the Matrix?

Nets and CCs HPWL

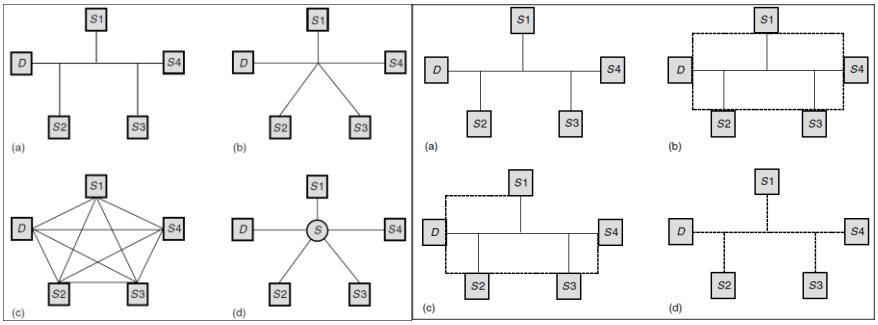
- Component C1
  - Connected to C2, C3



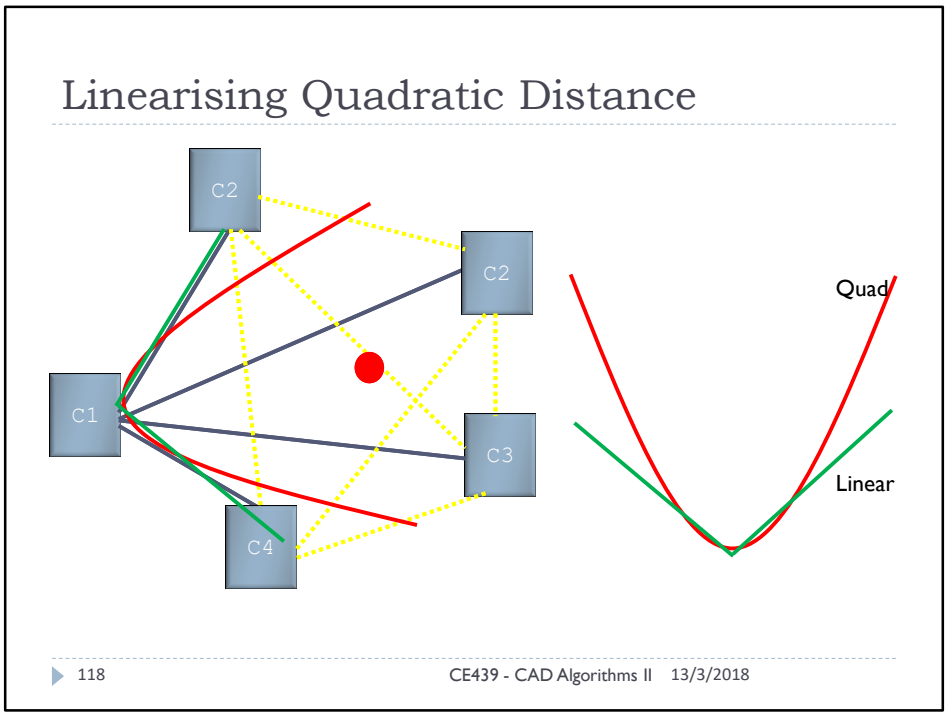
Laplacian matrix HPWL does not see nets net1, net2 as individual nets
Lumped HPWL is effectively used, except for individual net weights, which may be used, to compensate

Analytical Net Models

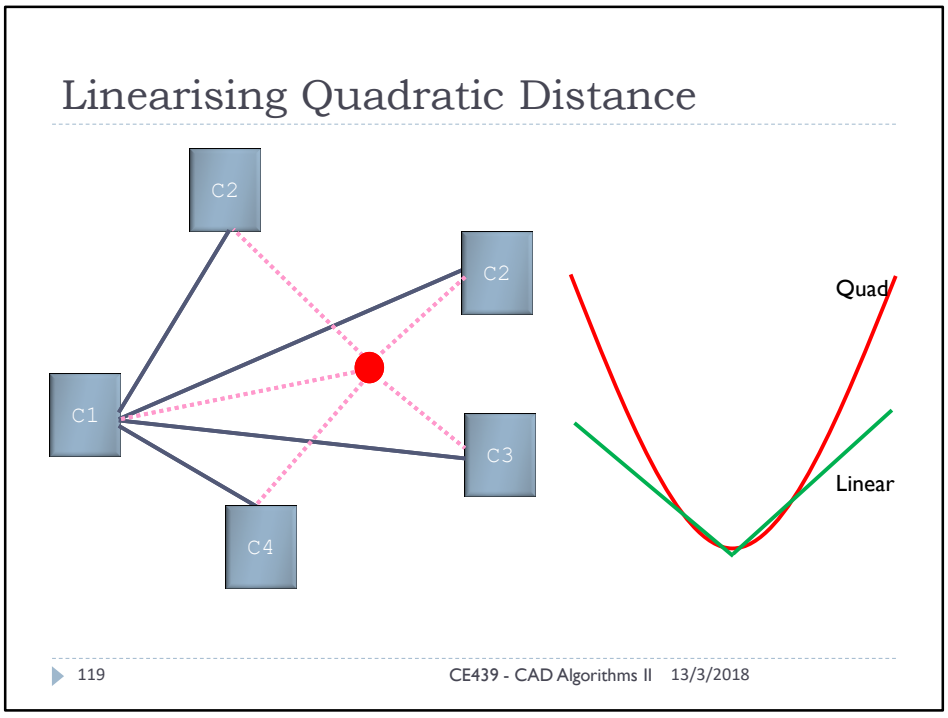
- Weights Applied Directly on the Laplacian Matrix



- LHS: (a) Net, (b) Hyperedge, (c) Clique, (d) Star
- RHS: (a) routed Net, BB Model (for HPWL), MST, Steiner Tree



In the P2P model, only output to input connections are modelled.
Thus, it is possible for the solution to be uneven lengths to output CCs.
Adding Clique or Star Points can alleviate this issue.
Linearisation can also be achieved by appropriate factoring of Quadratic lengths.



Star Model requires an Initial Placement!

Analytical Net Models

TABLE 17.1
Bounds on the Ratios of Different Net Models

	BB(V)	Steiner(V)	Clique(V)	Star(V)
BB(V)	1	1	1	1
Steiner(V)	$\frac{\lceil\sqrt{n-2}\rceil}{2}+\frac{3}{4}$	1	$\begin{cases}\frac{9}{8} & \text{for } n=4\\ 1 & \text{for } n\neq 4\end{cases}$	1
Clique(V)	$\frac{\lceil\frac{n}{2}\rceil\lfloor\frac{n}{2}\rfloor}{n-1}$	$\frac{\lceil\frac{n}{2}\rceil\lfloor\frac{n}{2}\rfloor}{n-1}$	1	1
Star(V)	$\lfloor\frac{n}{2}\rfloor$	$\lfloor\frac{n}{2}\rfloor$	$\frac{n-1}{\lceil\frac{n}{2}\rceil}$	1

The following result tells how well the other three net models approximate the length of an optimum rectilinear Steiner tree. For two-terminal nets, all the net models are identical.

Theorem 1 *Let V be a finite set of points in $\mathbb{R}^2$ and $n := |V| \geq 3$. Then Table 17.1 shows an upper bound on $\frac{\mathcal{M}_1(V)}{\mathcal{M}_2(V)}$ for net models $\mathcal{M}_1$ (row) and $\mathcal{M}_2$ (column) from BB, Steiner, Clique, and Star.*

As an example how to read Table 17.1, the entry in the second row and third column says that Steiner(V) $\leq \frac{9}{8}$ Clique(V) for all V and Steiner(V) $\leq$ Clique(V) if $n \neq 4$. All inequalities are essentially tight for all n. This result is due to Brenner and Vygen (2001).

- **Steiner(V) ≤ Clique(V) ≤ Star(V) for $n \neq 4$**
  - **Clique is superior to Star**
  - estimates the length of an optimum rectilinear Steiner tree more accurately

Typical net models include clique or star models, as shown in Figure 18.2 for a five-pin net. In the clique model, each *k*-pin net is replaced by *k(k* − 1*)/*2 two-pin nets. In the star model, a star node is added for each net to which all pins of the nets are connected. If the weight of a *k*-pin hyperedge is *W*, it is common to weight the set of two-pin nets using a weight such as *W/(k* − 1*)*

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Steiner(V)	$\frac{\lceil \sqrt{n-2} \rceil}{2} + \frac{3}{4}$	1	$\begin{cases} \frac{9}{8} & \text{for } n = 4 \\ 1 & \text{for } n \neq 4 \end{cases}$	1
Clique(V)	$\frac{\lceil \frac{n}{2} \rceil \lfloor \frac{n}{2} \rfloor}{n-1}$	$\frac{\lceil \frac{n}{2} \rceil \lfloor \frac{n}{2} \rfloor}{n-1}$	1	1
Star(V)	$\lfloor \frac{n}{2} \rfloor$	$\lfloor \frac{n}{2} \rfloor$	$\frac{n-1}{\lceil \frac{n}{2} \rceil}$	1

The following result tells how well the other three net models approximate the length of an optimum rectilinear Steiner tree. For two-terminal nets, all the net models are identical.

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Star Weight = k x Clique Weight, for k-size net

- Steiner(V) ≤ Clique(V) ≤ Star(V) for $n \neq 4$
  - BUT: Clique is *very compute intensive* as Matrix is dense
  - **Hybrid Net Model**
    - **For net size < 4 use Clique, >= 4 use Star**

Clique introduces k^2 non-zero additional entries in Laplacian Matrix
Star introduces k non-zero additional entries in Laplacian Matrix
Hybrid Model – see Fastplace placer – both models are proven equivalent for scaled clique weight

Typical net models include clique or star models, as shown in Figure 18.2 for a five-pin net. In the clique model, each *k*-pin net is replaced by *k(k* − 1*)/*2 two-pin nets. In the star model, a star node is added for each net to which all pins of the nets are connected. If the weight of a *k*-pin hyperedge is *W*, it is common to weight the set of two-pin nets using a weight such as *W/(k* − 1*)*

Analytical Net Models

- Weights Applied Directly on the Laplacian Matrix
- Clique Model
  - Utilizes all possible two-pin connections of a net
  - For a net with p vertices a complete graph is considered, where the weight of each edge is $\frac{1}{|p|-1}$ or $\frac{2}{|p|}$
  - Number of tree edges is (|p| – 1)
  - Number of clique graph edges is $\binom{n}{2} = \frac{n!}{2! \times (n-2)!} = \frac{1}{2}n(n-1)$
- Sum of Clique edges is model
- Thus, under these models, for a clique of size p we have
  - $\frac{2}{|p|}\frac{1}{2}|p|(|p|-1) = |p|-1$ or $\frac{1}{(|p|-1)}\frac{1}{2}|p|(|p|-1) = \frac{1}{2}|p|$

The clique model stems from the Analytical Placement Cost Function, which computes all point to point WL

Analytical Net Models

- Weights Applied Directly on the Laplacian Matrix
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  - Number of clique graph edges is: $\binom{n}{2} = \frac{n!}{2! \times (n-2)!} = \frac{1}{2}n(n-1)$
  - Quadratic Complexity on Net Size
- Star Model
  - Introduces an additional star pin per net (median)
  - Connects each net pin to the star pin
  - A net consisting of p vertices will be represented by $|p| - 1$ edges

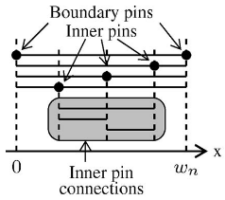
The notion of net models was introduced by Partitioning Algorithms.
If a net on $|e|$ vertices is represented as a complete graph on $|e|$ vertices and if $|e|$ is large, the $|e|$
vertices will likely be placed in the same block after bipartitioning. The result is that nets with small
numbers of vertices may be cut because of the predominance of large nets [SK72]. The solution to
this problem is to weight the graph edges of the net so that regardless of how vertices in the graph
are partitioned, the sum of the weights of the edges cut should be as close to 1 as possible. Generally,
it will not be possible for this sum to be exactly 1 as the theorem below states.

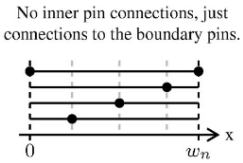
Bound2Bound Net Model

- Linear complexity – Matrix modification
- Net consists of one or more two-pin connections
- For each connection, weight is determined by:

$$w_{x,pq}^{\mathrm{B2B}} = \begin{cases} 0, & \text{if pin } p \text{ and pin } q \text{ are inner pins} \\ \frac{2}{P-1}\frac{1}{\left|x_p^{\mathrm{pin}} - x_q^{\mathrm{pin}}\right|}, & \text{else.} \end{cases}$$

Clique Model adds internal net lengths





Bound2Bound does not

Bound2Bound Net Model

- WL Quadratic to Linear Conversion through Bound2Bound Net Model: (left pin p=1, right pin q = 2)

$$\Gamma_{n,x} = \frac{1}{2}\sum_{p=1}^{P}\sum_{q=p+1}^{P} w_{x,pq}^{\mathrm{B2B}}\left(x_p^{\mathrm{pin}} - x_q^{\mathrm{pin}}\right)^2$$

$$= \frac{1}{2}\frac{2}{P-1}\left[\left|x_1^{\mathrm{pin}} - x_2^{\mathrm{pin}}\right| + \sum_{q=3}^{P}\left|x_1^{\mathrm{pin}} - x_q^{\mathrm{pin}}\right| + \sum_{q=3}^{P}\left|x_2^{\mathrm{pin}} - x_q^{\mathrm{pin}}\right|\right]$$

$$= \frac{1}{P-1}\left[w_n + (P-2)w_n\right]$$

$$= w_n.$$

$$w_{x,pq}^{\mathrm{B2B}} = \begin{cases} 0, & \text{if pin } p \text{ and pin } q \text{ are inner pins} \\ \frac{2}{P-1}\frac{1}{\left|x_p^{\mathrm{pin}} - x_q^{\mathrm{pin}}\right|}, & \text{else.} \end{cases}$$

$$w_n = \left|x_1^{\mathrm{pin}} - x_2^{\mathrm{pin}}\right|$$

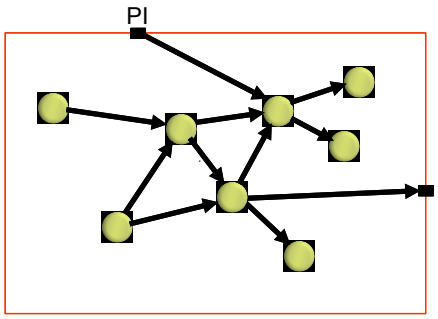
Furthermore, all possible two pin connections are separated into the following three categories:

1. connections between the two boundary pins (p = 1, q = 2),
2. connections between the "left" boundary pin 1 and the inner pins (p = 1, $q \geq 3$),
3. and connections between the "right" boundary pin 2 and the inner pins (p = 2, $q \geq 3$).

The inner two-pin connections ($p \geq 3$, $q > 3$) are not considered, as they have a connection weight of zero

Incremental HPWL Computation

- For minor HPWL changes, it is best to incrementally update HPWL



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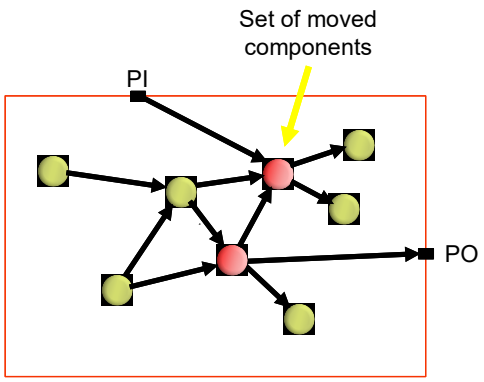

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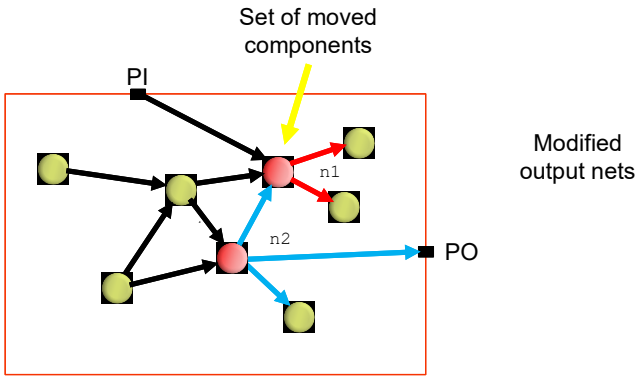

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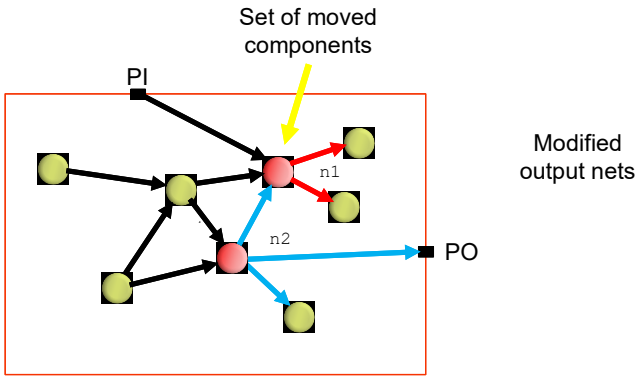

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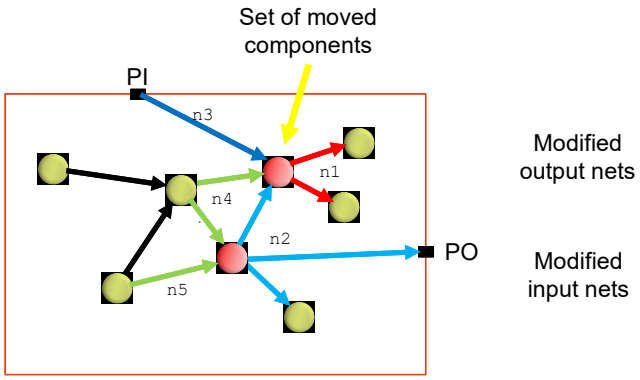


Placement Density Map as a Analytical Force Directed Method

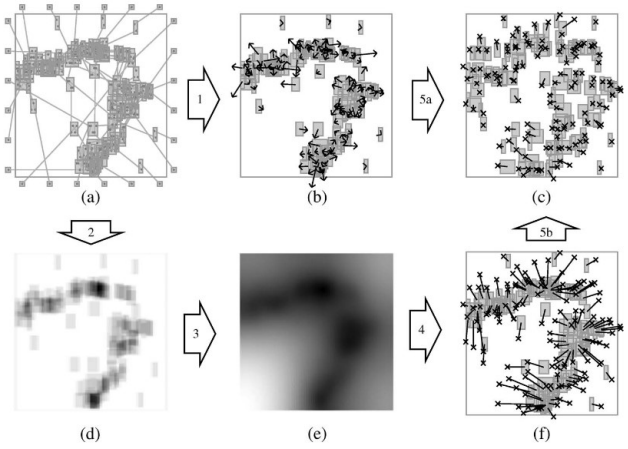


Fig. 6. Illustration of one placement iteration. The numbers in the big arrows represent the sequence of the steps executed in each placement iteration. (d), (e) Density plots, with white and black colors representing low and high densities, respectively. (a) Starting placement. (b) Hold force. (c) Resulting placement. (d) Supply and demand system D. (e) Potential Φ. (f) Target points and move force.

Understanding Density Map Properties

- **Density(Bin Area) = Density(Bin Util.) x (Binw * Binh)**
- **Gradient(Bin Area) = Gradient(Bin Util.) x (Binw * Binh)**

binw	1
binh	1
util.	0,7

Density Map (D - S) must be balanced
To balance, Component Area must be multiplied by (1/utilisation)!
Default (empty) Supply, per bin is -1

	Density	Gradient X	Gradient Y
Utilisation	-1 -1 -1 / -1 2 2 2 -1 / -1 3 5 3 -1 / -1 1 2 1 1 / -1 -1 -1	1,5 0 -1,5 / 3 0 -3 / 1,5 0 -0,5	2 3 2 / -0,5 0 -0,5 / -2 -3 -2
Area	-1 -1 -1 / -1 2 2 2 -1 / -1 3 5 3 -1 / -1 1 2 1 1 / -1 -1 -1	1,5 0 -1,5 / 3 0 -3 / 1,5 0 -0,5	2 3 2 / -0,5 0 -0,5 / -2 -3 -2
Check	Density(Bin Area) = Density(Bin Util.) x (Binw * Binh) Gradient(Bin Area) = Gradient(Bin Util.) x (Binw * Binh)	0 0 0 / 0 0 0 / 0 0 0	0 0 0 / 0 0 0 / 0 0 0

Density Map – 1

- The distribution of the cells is modeled by a Demand-Supply System D.

$$\boldsymbol{D(x,y) = D^{dem}(x,y) - D^{sup}sup(x,y)}$$

- The $D^{dem}(x,y)$ refers to the cells and the $D^{sup}(x,y)$ refers to the placement area (usually the core)

- Cells are moved away from high density regions (high *DEMAND*) to low-density regions (high *SUPPLY*)

Density Map - 2

- Due to the fact that $D^{dem} < D^{sup}$, the system **D** must be balanced and adapted as it is presented, below:

$$\iint_{-\infty}^{+\infty} D^{dem}(x,y)dxdy = \iint_{-\infty}^{+\infty} D^{sup}(x,y)dxdy$$

- To formulate the demand, a rectangle function **R** is used:

$$\boldsymbol{R(x,y,x_{ll},y_{ll},w,h)} = \begin{cases} \mathbf{1}, & \boldsymbol{if}\ \mathbf{0} \le \boldsymbol{x} - \boldsymbol{x_{ll}} \le \boldsymbol{w} \\ & \boldsymbol{\wedge}\ \mathbf{0} \le \boldsymbol{y} - \boldsymbol{y_{ll}} \le \boldsymbol{h} \\ \mathbf{0}, & \boldsymbol{elsewhere} \end{cases}$$

  - where,
    - $\boldsymbol{x}, \boldsymbol{y}$ represent all the points inside the rectangle
    - $\boldsymbol{x_{ll}}, \boldsymbol{y_{ll}}$ represent either the center or the left corner of a module/cell
    - $\boldsymbol{w}, \boldsymbol{h}$ represent the width and height of the cell, accordingly.

Density Map - 3

- Cell i demand

$$D_{cell}^{dem}(x,y) = d_{cell,i} * R(x, y, x'_i - \frac{w_i}{2}, y'_i - \frac{h_i}{2}, w_i, h_i)$$

- The individual module density $d_{cell,i}$ is usually set to **1**.

- Cell i supply

$$D_{cell}^{sup}(x,y) = d_{sup} * R(x, y, x_{chip}, y_{chip}, w_{chip}, h_{chip})$$

- Supply density:

$$d_{sup} = \sum_{i=1}^{M+F} \frac{(d_{cell,i} * A_m)}{A_{chip}}$$

Density Map - 4

- **D** is interpreted as a charge distribution and creates an electrostatic potential φ, based on Poisson's equation:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) * \varphi(x,y) = -D(x,y)$$

- Solving the equation determines target points

$$\dot{x}_i = x'_i - \frac{\partial}{\partial x}\varphi(x,y)\Big|_{(x'_i,y'_i)}$$

previous location

- Potential φ Gradients are collected in a vector

$$\Phi = \left(\left(\partial/\partial x\right)\varphi\Big|_{(x'_1,y'_1)}, \left(\partial/\partial x\right)\varphi\Big|_{(x'_2,y'_2)}, \ldots, \left(\partial/\partial x\right)\varphi\Big|_{(x'_M,y'_M)} \right)^T$$

Density Map - 5

- correct preset supply value is indeed -1
  - i.e. negative bin unit supply
- Demand was NOT originally adapted to the Supply
  - When this is the case, the two surface integrals are not identical, i.e. there is more Supply than Demand
  - This will lead to an uneven poisson solution, particularly for Dirichlet boundary conditions, where the "more empty space" will create deep troughs (double integral of space produces potential, so maximises at its centre)
    - aa a consequence causes spreading saturation.
  - This behaviour was indeed observed when using Dirichlet boundary constraints
  - Neumann boundary constraints do not exhibit this behaviour, due to the explicit gradient (zero) constraint.

Density Map - 6

- to adapt demand to supply, totalsupply = totaldemand
  - component Area must be inflated by (1/utilisation)
  - as utilisation = demand/supply
  - demand = supply x 1/utilisation;
- The (1/utilization) area scaling factor can then be converted to
  - a dimensions factor for components, as 1/SQRT(utilisation)

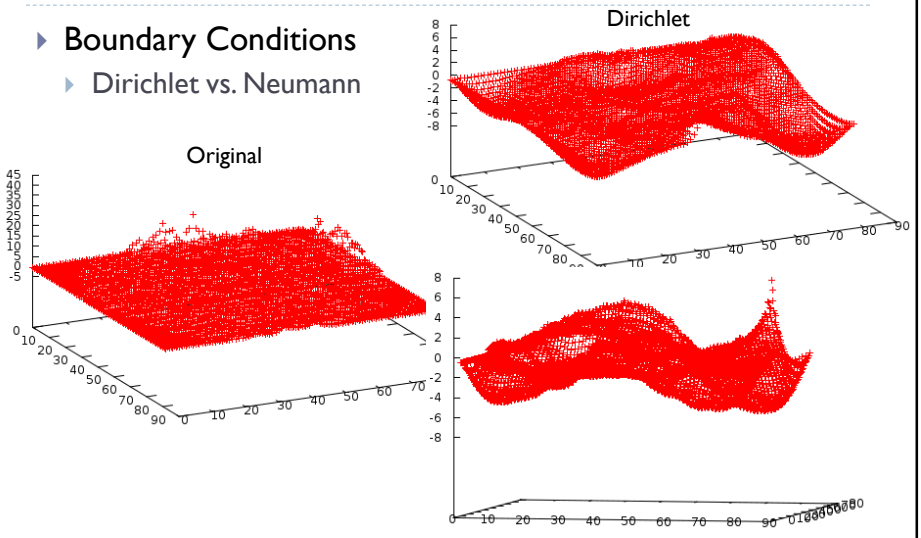
```
utilisationwhscalingfactor = 1/sqrt(utilisation);
// dimensions, i.e. width, height scaling factor for demand
supply balancing //
```

```
cw = cw * utilisationwhscalingfactor;
ch = ch * utilisationwhscalingfactor;
// scale component, cluster BB width, height by
utilisationwhscalingfactor, to adapt demand to supply //
```


// demand must be adapted to supply, so that their two surface integrals are identical, and Demand - Supply is balanced //
// Demand - Supply is interpreted as charge density, and is integrated twice to yield (1) force and (2) potential (work/unit charge) //

Density Map – 7

- Boundary Conditions
  - Dirichlet vs. Neumann



Quality Control - Acceleration

74 4 Kraftwerk: A Fast and Robust Quadratic Placer Using an Exact Linear Net Model

after loading the placement of the original circuit at different iterations: 5, 10, 15, 20, and 25. The results of Table 4.1 reveals that ECO can be applied at various iterations without harming the net length (increase below 0.1%) but with great improvement in the CPU time (up to 80%) compared if the changed circuit is placed from scratch.

4.4.2 Quality Control

In order to control the important trade-off between the quality of placement and the CPU time, a quality control procedure is called at the end of each iteration in global placement (see Figure 4.4).

This trade-off presents a challenge in everyday placement usage. On the one hand the deadline for placement can be near and therefore the chip has to be placed in short CPU time. On the other hand the quality of placement can be very important with no limit of CPU time.

Equation (4.11) shows that the quadratic cost function Γ consists of two-pin connections and each two-pin connection has the weights $w_{e,x}$ and $w_{e,y}$. These connection weights are changed in each iteration of global placement in order to adapt the quadratic cost function Γ to a realistic objective, e.g., to the net length measured in HPWL or to fulfill timing requirements. Section 4.2.2 describes the Bounding-Box net model, which uses the connection weights in order to model the HPWL in the quadratic cost function Γ. The authors of [29] show a technique to modify the connection weights in order to model timing requirements in the quadratic cost function Γ.

Hence the more iterations are spent in global placement, the better is the modeling of the real objective and the higher is the quality. On the other hand, the more iterations the global placement needs, the higher is the CPU time, since every single iteration takes a fix CPU time.

If the average module movement μ is controlled to be μ_T in every iteration, then the iteration count l_{global} of global placement is indirectly proportional to μ_T. This is because each module has to move a certain length Λ in global placement in order to get an overlap-free placement and thus following holds true:

$$\Lambda \approx \mu_T \cdot l_{global} \Leftrightarrow l_{global} \propto \frac{1}{\mu_T}. \tag{4.27}$$

To control the module movement μ to be μ_T, the target points' spring constants $\hat{w}_i$ are used, since a target point with a small spring constant attracts its module less than with a high spring constant.

Altogether, a certain target movement μ_T is set for the quality control and the average module movement μ is compared to μ_T in each iteration: if $\mu < \mu_T$, then every $w_{T,i}$ is increased and if $\mu > \mu_T$ then every $w_{T,i}$ is decreased. Details about this scaling process and the spring constants of the target points in general are explained in Sect. 4.4.3

Figure 4.5 shows that the presented quality control can efficiently govern the important trade-off between quality of placement and CPU time by using its only

4.4 Implementation Details 75

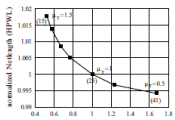


Fig. 4.5. Trade-off between quality of placement, measured in HPWL net length, and CPU time with quality control's parameter μ_T. The results are based on six circuits of the ISPD 2005 contest benchmarks. The values in the brackets express the average of iteration count l_{global}.

parameter μ_T. Here the quality of placement is measured in the HPWL of all nets. Compared to $\mu_T = 20$, the CPU time can be decreased to 50% at $\mu_T = 30$. At this point, the quality of placement is less than 2% worse than at the starting point. On the other side at $\mu_T = 10$, the quality can be improved by around 0.5% at a CPU time increase of 70%. With a quality range of less than 2% and a CPU time range more than 100%, Figure 4.5 also demonstrates that our placement algorithm is very robust in quality of placement but flexible in CPU time.

4.4.3 Spring Constants of the Target Points

Please note that the following detailed description of the target points' spring constants does not affect the proof of convergence in Sect. 4.3.2 since this proof is independent of the target points' spring constant.

The spring constants of the target points $\hat{w}_i$ are initialized with

$$\hat{w}_i = \frac{1}{M} \qquad M\text{: Number of movable modules} \tag{4.28}$$

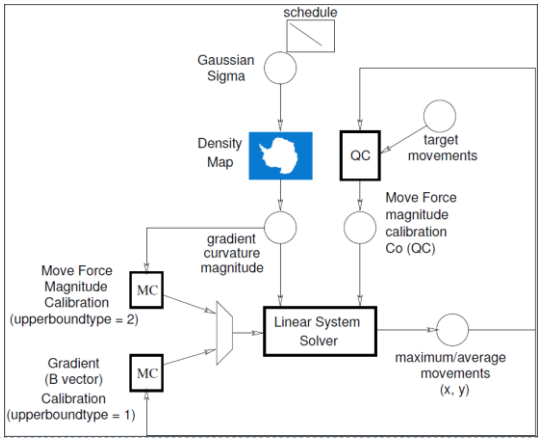
Then a function $\kappa(\mu)$ is used in every iteration to scale the spring constants $\hat{w}'_i$ of the last iteration to the spring constants $\hat{w}_i$ in the current iteration depending on the module movement μ, i.e., depending on the change of the modules positions during the last iteration:

$$\hat{w}_i = \kappa(\mu) \cdot \hat{w}'_i. \tag{4.29}$$

Figure 4.6 shows this scaling function $\kappa(\mu)$: At a smaller module movement μ than the target module movement μ_T, the scaling function $\kappa(\mu)$ is greater than one. With (4.29) the spring constants of the target points are increased. At a higher module movement μ than the target movement μ_T, the scaling function $\kappa(\mu)$ is smaller than one and the spring constants of the target points are decreased.

Quality Control - Acceleration

- GP Algorithm Acceleration
  - Iteratively Calibrates Move Force to achieve a target movement
    - **In addition, uses heuristic move weight scaling in first iteration**



Quality Control - Acceleration

- `// alternative, iterative QC weight scaling, i.e. w(i) = qcfactor x w(i - 1) //`
- `// Update KW2weightmatrix_X, KW2weightmatrix_Y weight values // // updated weight = (w x h) x 1/KW2avgcellarea x qcmultiplier x weightmultiplier //`
- `KW2weightmatrix_X[i] = min(maximumweight, KW2weightmatrix_X[i] * qcmultiplierx); // maximum weight global is upper weight bound //`
- `KW2weightmatrix_Y[i] = min(maximumweight, KW2weightmatrix_Y[i] * qcmultipliery); // maximum weight global is upper weight bound //`
- The interaction between the two, provided the bin size is set to the average cell area and combined with Neumann boundary conditions for the Density Map, achieve a good HPWL result with a small number of GP iterations
- The iterative QC weight scaling, combined with the initial weight scaling, manage to accelerate the GP process around a larger value of target movements, without a significant loss in HPWL QOR
- Neumann boundary is necessary, as gradient values will scale gracefully with design size, as **Density Map height (initial) does not confine gradient values to smaller Density Map area**

The interaction between the two, provided the bin size is set to the average cell area and combined with Neumann boundary conditions for the Density Map, achieve a good HPWL result with a small number of GP iterations. The iterative QC weight scaling, combined with the initial weight scaling, manage to accelerate the GP process around a larger value of target movements, without a significant loss in HPWL QOR. Neumann boundary is necessary, as gradient values will scale gracefully with design size, as Density Map height does not confine gradient values to smaller Density Map area.

Density @ centre = (D – 0)/Dx = (N/(N/Dx) – 0)/Dx = ~1

Quality Control – Initial Iteration

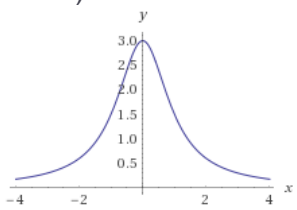
- Weight multiplier
  - 1/(total number of placeable elements)/(average of bin width, bin height) is used to normalise, *i.e.* scale down the bin utilisation gradient value for the initial GP iteration
- Scaling avoids excess weight values being produced from the large initial bin *utilisation gradient* values
  - the latter are large due to placeable element stacking at the center of the Core Area, particularly for the **B2B/B2B** Net Model
    - no scaling would result in many upper movement bound violations in early iterations
- In scaling factor
  (total number of placeable elements)/(average of bin width, bin height)
  - the numerator corresponds to the maximum bin demand at the centre of the core area for the B2B/B2B Net Model
  - this is identical to bin utilisation, if bin area corresponds to the average cell area, *i.e.* MAX(bin demand);
- As for the gradient value computation, neighbouring bins are used, thus **dx** used in the bin utilization gradient **dy/dx** is bin width or height
  - therefore, the gradient scaling factor value is **MAX(bin demand)/average dx**.

The weight multiplier 1/(total number of placeable elements)/(average of bin width, bin height) is used
to normalise, i.e. scale down the bin utilisation gradient value for the initial GP iteration; scaling
avoids excess weight values being produced from the large initial bin utilisation gradient values; the
latter are large due to placeable element stacking at the center of the Core Area, particularly for the
B2B/B2B Net Model; no scaling would result in many upper movement bound violations in early iterations.
In scaling factor (total number of placeable elements)/(average of bin width, bin height), the numerator
corresponds to the maximum bin demand at the centre of the core area for the B2B/B2B Net Model; this is
identical to bin utilisation, if bin area corresponds to the average cell area, i.e. MAX(bin demand);
as for gradient value computation, neighbouring bins are used, the dx used in the gradient dy/dx of bin
utilisation is bin width or height; therefore, the scaling factor value is MAX(bin demand)/average dx.

Density @ centre = (D – 0)/Dx = (N/(N/Dx) – 0)/Dx = ~1

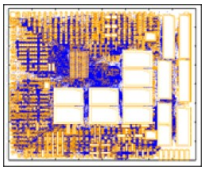
Quality Control - Acceleration

- Original Acceleration Function
  - 1 + tanh(log(1/x))
- Introduced Polynomial Acceleration which is more versatile
  - n/(x^2+1), where n is a weight factor
  - Example on RHS is with n=3
  - 1 + tanh(log(1/x)) is equivalent to 2/(x^2+1)
- Default is 2/(x^2+1)
  - User may specify weight, if needed

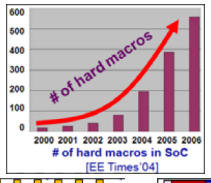


Modern Placement Challenges

- High complexity
  - Millions of objects to be placed
- Placement constraints
  - Preplaced blocks
  - Chip density, etc.
- Mixed-size placement
  - Hundreds/thousands of large macros with millions of small standard cells
- Many more
  - Datapath
  - 3D IC
  - Analog, etc.

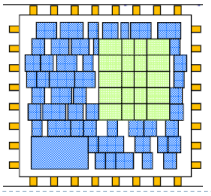


2.5M placeable objects mixed-size design

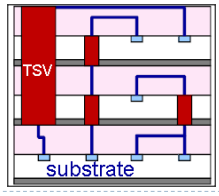


Macros in SoC Design

Datapath Placement

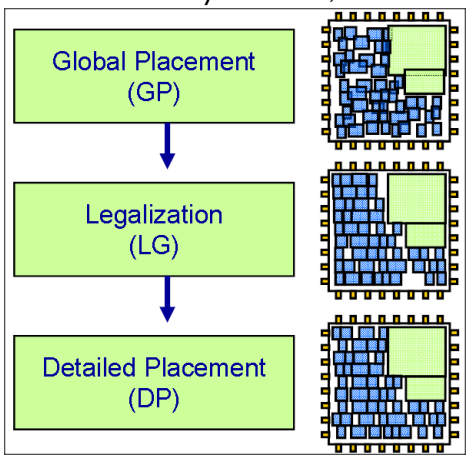


3DIC



Typical Modern Placement Flow

- Chen, et al., "A high quality analytical placer considering preplaced blocks and density constraint," ICCAD-06 (TCAD-08)



Computes the best position for each object to minimize the cost, e.g. WL, ignoring overlaps

Removes all overlaps between objects

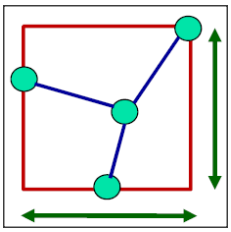
Improves the Legalised solution cost further

HPBB Wirelength Models

- Linear (Golden) HPBB Model:

$$W(x,y) = \sum_{e \in E} \left(max_{v_i,v_j \in E} |x_i - x_j| + max_{v_i,v_j \in E} |y_i - y_j| \right)$$

  - Is neither smooth or differentiable
  - Approximations
    - Quadratic (Gordian)
    - Lp-Norm
    - LSE (Log-Sum-Exponential), *i.e.* Soft Max Approximation
    - CHKS, etc.



Log-Sum-Exp (LSE) Wirelength Model

- Soft Max Definition
  - The soft maximum of two variables x, y is the function:

$$g(x, y) = \ln(e^x + e^y)$$

- The soft maximum approximates the hard maximum and is a convex function just like the hard maximum (max)
  - the accuracy of the soft maximum depends on scale, e.g. g(1, 2) = 2.31, but g(10, 20) = 20.00004
  - The *hardness* of the soft maximum can be controlled by a parameter k:

$$g(x, y) = \ln\left(e^{(kx)} + e^{(ky)}\right)/k$$

To see that the soft maximum approximates the hard maximum, note that if x is a little bigger than y, exp(x) will be a lot bigger than exp(y). That is, exponentiation exaggerates the differences between x and y. If x is significantly bigger than y, exp(x) will be so much bigger than exp(y) that exp(x) + exp(y) will essentially equal exp(x) and the soft maximum will be approximately log(exp(x)) = x, the hard maximum.

Log-Sum-Exp (LSE) Wirelength Model

- Introduced by Naylor in 2001

$$W_{LSE}(x,y) = \gamma \sum_{e \in E} (ln \sum_{v_i \in E} e^{\frac{x_i}{\gamma}} + ln \sum_{v_i \in E} e^{\frac{-x_i}{\gamma}} + ln \sum_{v_i \in E} e^{\frac{y_i}{\gamma}} + ln \sum_{v_i \in E} e^{\frac{-y_i}{\gamma}})$$

- whereby:

$$\max(x) \cong \gamma \ln \sum_{v_i \in E} e^{\frac{x_i}{\gamma}} \qquad -\min(x) = \max(-x)$$

- effective smooth, differentiable approximation for HPWL
- Approaches exact HPWL when $\gamma \rightarrow 0$
- **Has dominated modern placement for 10+ years!**

Weighted-Average (WA) Model

- Hsu, Chang, Balabanov, DAC 2011

$$W_{WA}(x,y) = \sum_{e \in E} \left(\frac{\sum_{v_i \in e} x_i \, e^{\frac{x_i}{\gamma}}}{\sum_{v_i \in e} e^{\frac{x_i}{\gamma}}} - \frac{\sum_{v_i \in e} x_i \, e^{\frac{-x_i}{\gamma}}}{\sum_{v_i \in e} e^{\frac{-x_i}{\gamma}}} + \frac{\sum_{v_i \in e} x_i \, e^{\frac{y_i}{\gamma}}}{\sum_{v_i \in e} e^{\frac{y_i}{\gamma}}} - \frac{\sum_{v_i \in e} x_i \, e^{\frac{-y_i}{\gamma}}}{\sum_{v_i \in e} e^{\frac{-y_i}{\gamma}}} \right)$$

- Ratio-ed LSE
- Weighted average of a set of x coordinates, xi, of a net i:
- X(xi) approximates the maximum value of xi:

$$X(x_i) = \frac{\sum_{v_i \in E} x_i F(x_i)}{\sum_{v_i \in E} F(x_i)} = \frac{\sum_{v_i \in E} x_i \, e^{\frac{x_i}{\gamma}}}{\sum_{v_i \in E} e^{\frac{x_i}{\gamma}}}$$

- effective smooth, differentiable approximation for HPWL
- Approaches exact HPWL when $\gamma \rightarrow 0$

Other Smooth Wire Models

- Quadratic Model: BonnPlace, FastPlace, Kraftwerk, mFAR
  - Not exact Wirelength Modelling

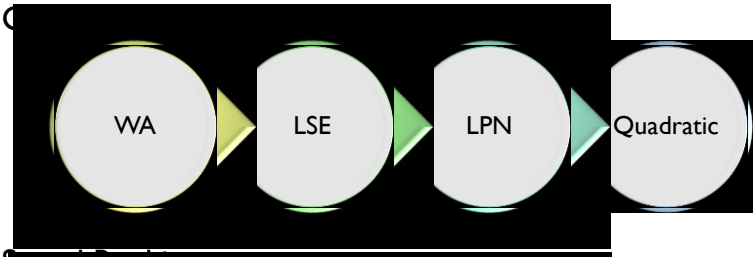
$$W_{QP}(x,y) = \sum_{e \in E} \Big(\sum_{v_i, v_j \in e, i<j} w_{ij}(x_i - x_j)^2 + \sum_{v_i, v_j \in e, i<j} w_{ij}(y_i - y_j)^2 \Big)$$

- Lp-Norm Model
  - Approaches exact HPWL when p → ∞
  - Is compute intensive

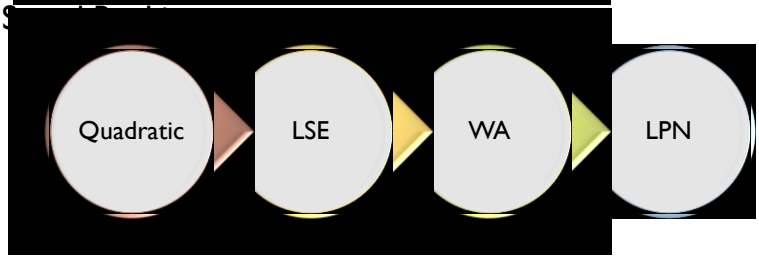
$$W_{LPN}(x,y) = \sum_{e \in E} \Big(\sqrt[p]{\sum_{v_i \in E} x_i^p} - \sqrt[p]{\sum_{v_i \in E} x_i^{-p}} + \sqrt[p]{\sum_{v_i \in E} y_i^p} - \sqrt[p]{\sum_{v_i \in E} y_i^{-p}} \Big)$$

Wire Models Ranking

- C

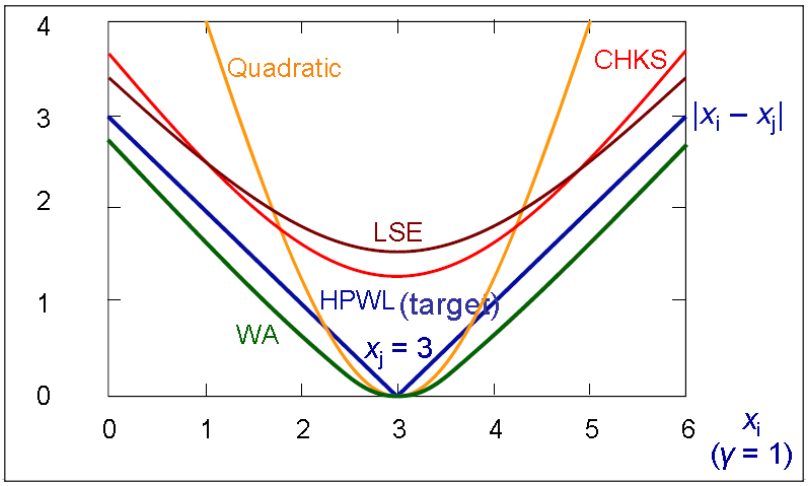


- S



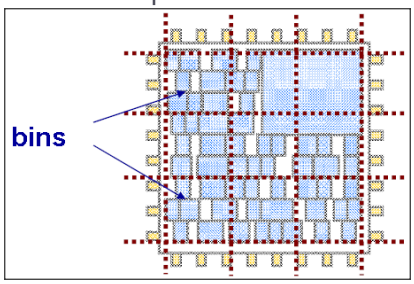
Wireload Models Comparison

- Y-axis is Wirelength – smooth 2 variable functions



Placement with Density Constraint

- Given the chip region and block dimensions, divide placement region into bins
- Determine (x, y) for all movable blocks
  - min W(x, y) -- wirelength function s.t.
  - 1 Density*b*(x, y) ≤ MaximumDensity*b*, for each bin *b*
  - 2. No overlap between blocks



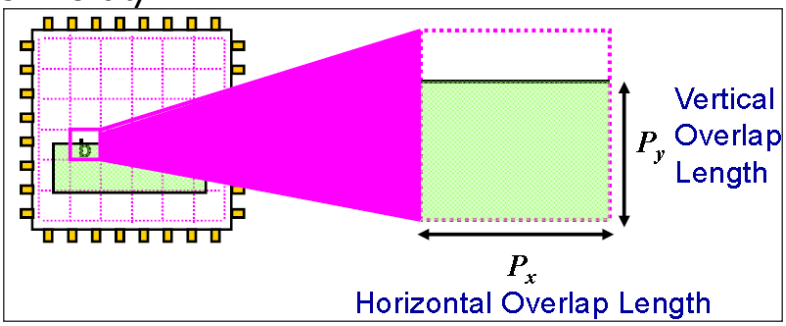
Density Definitions:

$$Density(Global) = \frac{\sum Area_{stdcells}}{\sum Area_{bins}}$$

$$Density(bin\ b) = \frac{\sum_{bin\ b} Area_{stdcells}}{Area_{bin\ b}}$$

Density Model

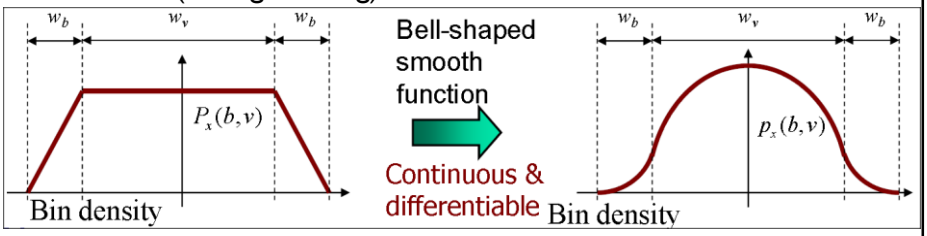
- Compute the block area of each bin to obtain the bin density



- Bin Density: $D_b(x,y) = \sum_{v \in V} P_x(b,v) P_y(b,v)$

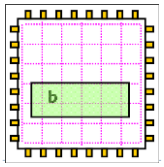
Density Smoothing

- Apply bell-shaped function to make bin density function smooth (Kahng & Wang)



$$D_b(x,y) = \sum_{v \in V} P_x(b,v) P_y(b,v)$$

$$D'_b(x,y) = \sum_{v \in V} c_v p_x(b,v) p_y(b,v)$$



$$p_x(b,v) = \begin{cases} 1 - ad_x^2, & 0 \le d_x \le \frac{w_v}{2} + w_b \\ b(d_x - \frac{w_v}{2} - 2w_b)^2, & \frac{w_v}{2} + w_b \le d_x \le \frac{w_v}{2} + 2w_b \\ 0, & \frac{w_v}{2} + 2w_b \le d_x, \end{cases}$$

where

$$a = 4/((w_v + 2w_b)(w_v + 4w_b))$$
$$b = 2/(w_b(w_v + 4w_b)),$$

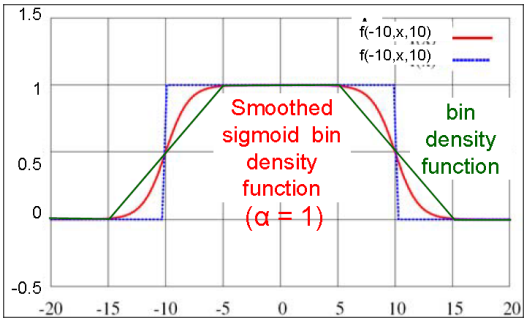
wb bin width, *wv* block width, *cv* normalization factor

Density Smoothing - Sigmoid

- $f(l, x, u) = l$, *if* $l < x < u$

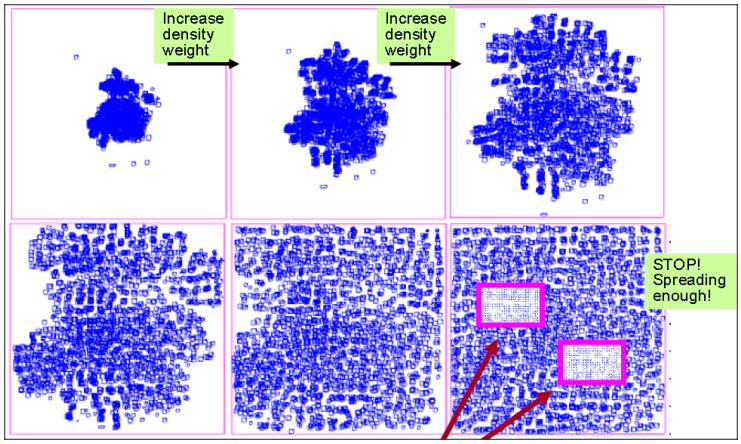
$$p(t) = \frac{1}{1 + e^{-\alpha t}}$$

- $f(l, x, u) \sim p(x - l)p(u - x)$



- effective smooth, differentiable approximation for the bin density function
- Approximates exact 0-1 logic function when α → 0

Density-based Placement Process



Analytical Placement Model

- Global Placement Problem with Density Constraint

$$\min W(x, y)$$
$$s.t.\, D_b(x, y) \leq M_b$$

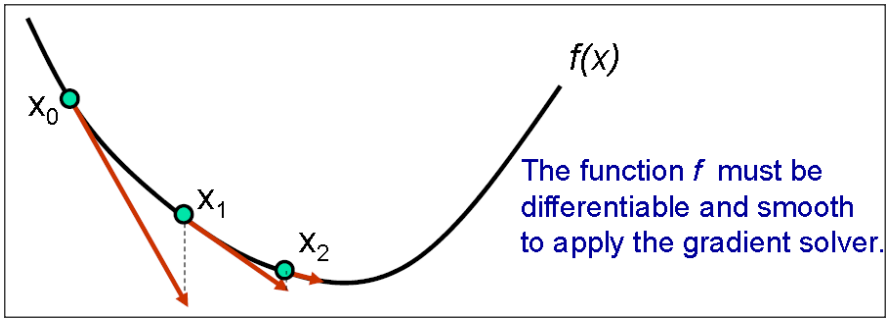
  - Minimise Wirelength, Db: density, Mb: max density of bin b
- Constraints can be relaxed into the Objective Function

$$\min W(x, y) + \lambda \sum_{bins\ b} \left(D_{b(x,y)} - M_b\right)^2$$

  - Use gradient methods to solve it
  - Increase lambda gradually to reduce density penalty and find optimal (x, y) under density constraint

Gradient Solvers

- min f(x)
- x0 ← initial value
- Repeat until convergence
  - $x_{i+1} = x_i - f'(x)|_{x=xi}$ * stepsize

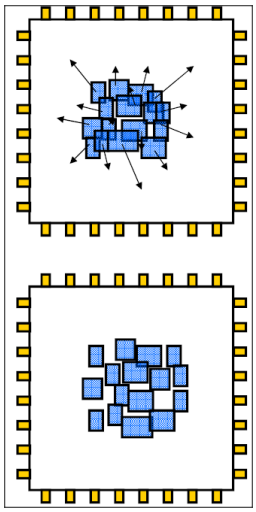


Dynamic Step-Size Control

- Step size is too large
  - May not converge to a good solution
- Step size is too small
  - Incur long running time
- Adjust the step size s.t. the **average Euclidean movement of all blocks is a fixed value**

$$stepsize\ \alpha_k = \frac{s}{\|d_k\|_2}$$

  - dk is conjugate direction(s)
  - s is a user-specified factor



Norm 2 is Euclidean Distance

Look-ahead Legalisation

Modern Academic Analytical Placers

Placer	Wirelength Model	Overlap Model	Integration	Optimization
APlace	LSE	Density	Penalty Method	Nonlinear
BonnPlace	Quadratic	Partitioning	Region Constraints	Quadratic
DPlace	Quadratic	Diffusion	Fixed Point	Quadratic
FastPlace	Quadratic	Cell Shifting	Fixed Point	Quadratic
FDP	Quadratic	Density	Fixed Point	Quadratic
Gordian	Quadratic	Partitioning	Region Constraint	Quadratic
hATP	Quadratic	Partitioning	Region Constaint	Quadratic
Kraftwerk2	Bound2Bound	Density	Fixed Point	Quadratic
mFAR	Quadratic	Density	Fixed Point	Quadratic
mPL6	LSE	Density	Penalty Method	Nonlinear
NTUPlace3	LSE/WA	Density	Penalty Method	Nonlinear
RQL	Quadratic	Cell Shifting	Fixed Point	Quadratic
SimPL	Bound2Bound	Partitioning	Region Constraint	Quadratic
Vasstu	LSE	Fixed Point	Fixed Point	Nonlinear