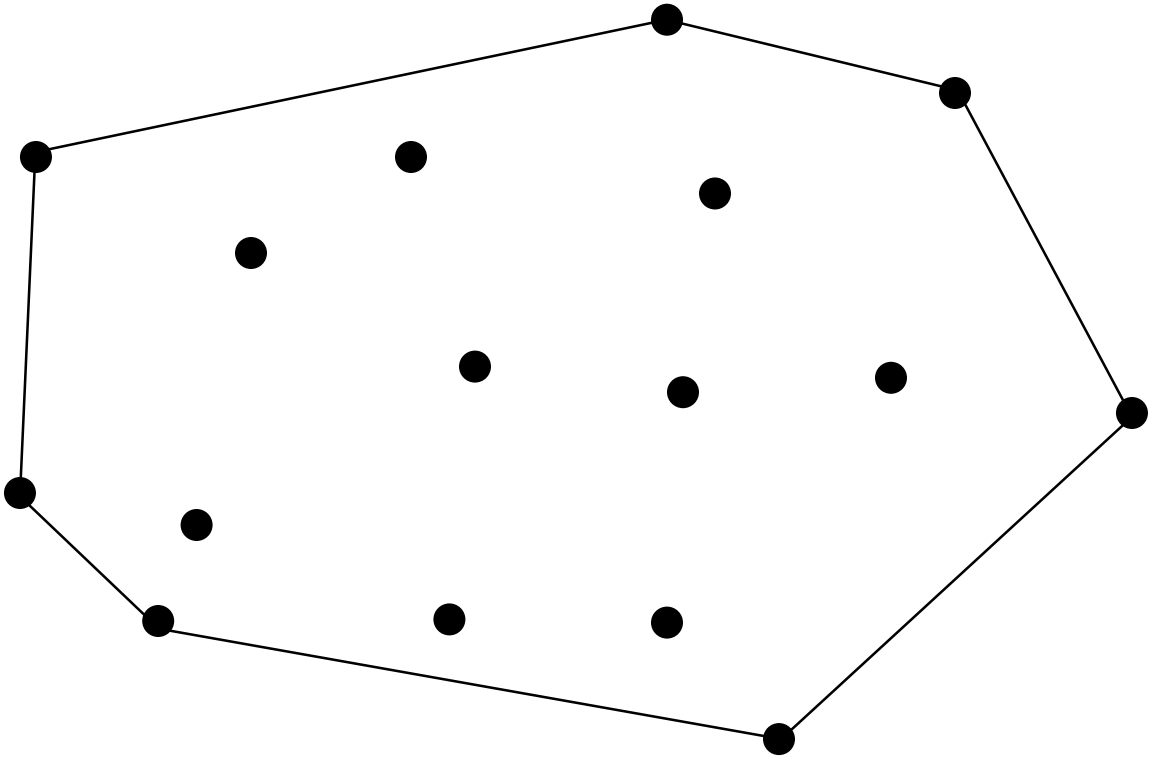
Computational Geometry

Convex Hull

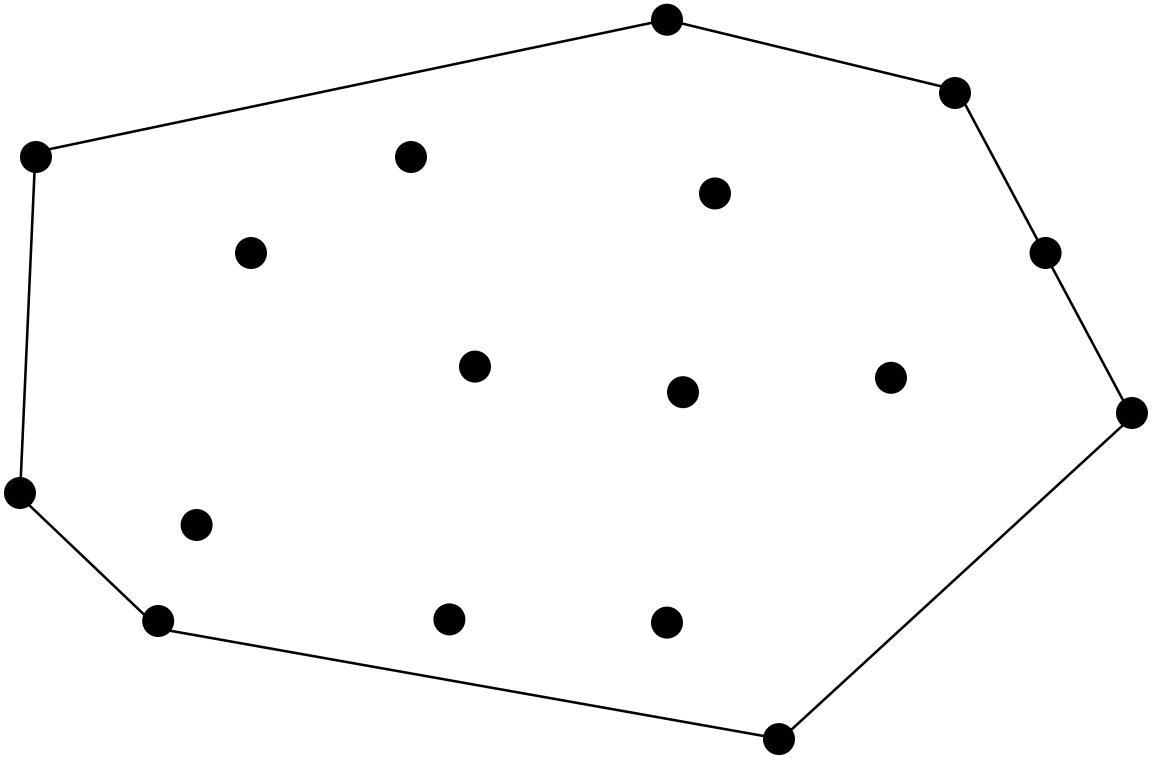
Convex Hull

- Given a set of n points, find the minimal convex polygon that contains all the points



Convex Hull Representation

- The convex hull is represented by all its points sorted in CW/CCW order
- Special case: Three collinear points

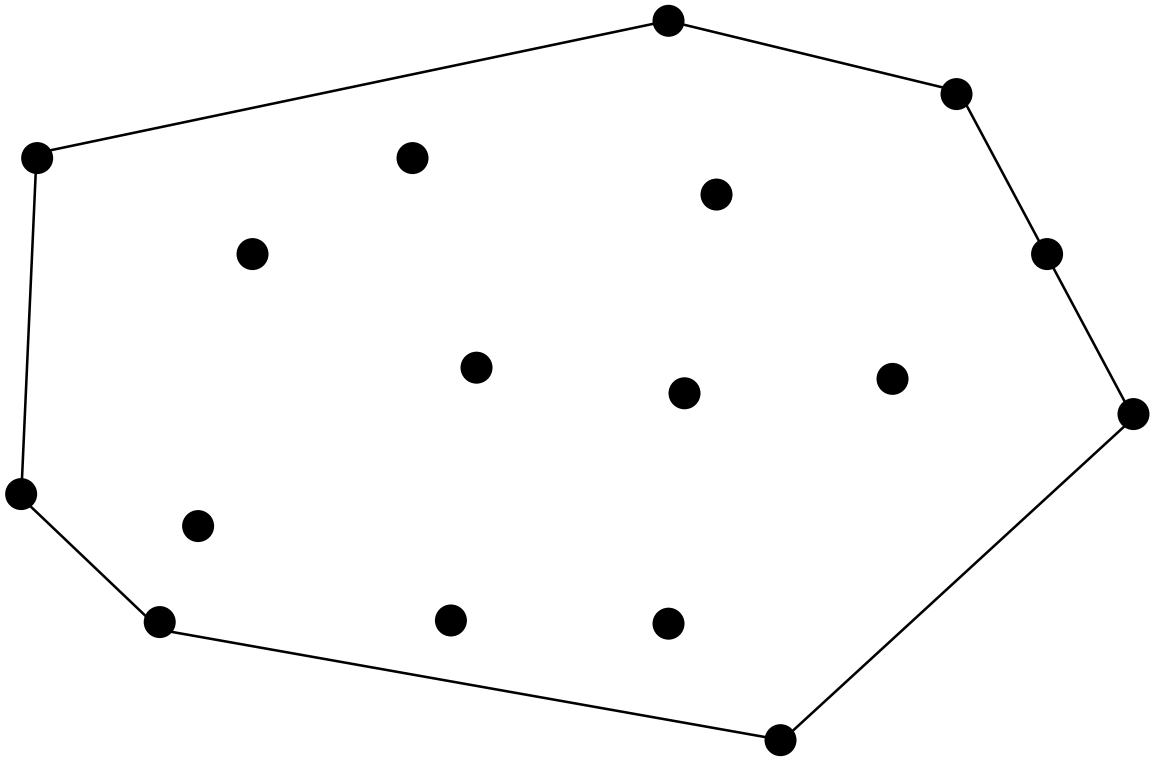


Naïve Convex Hull Algorithm

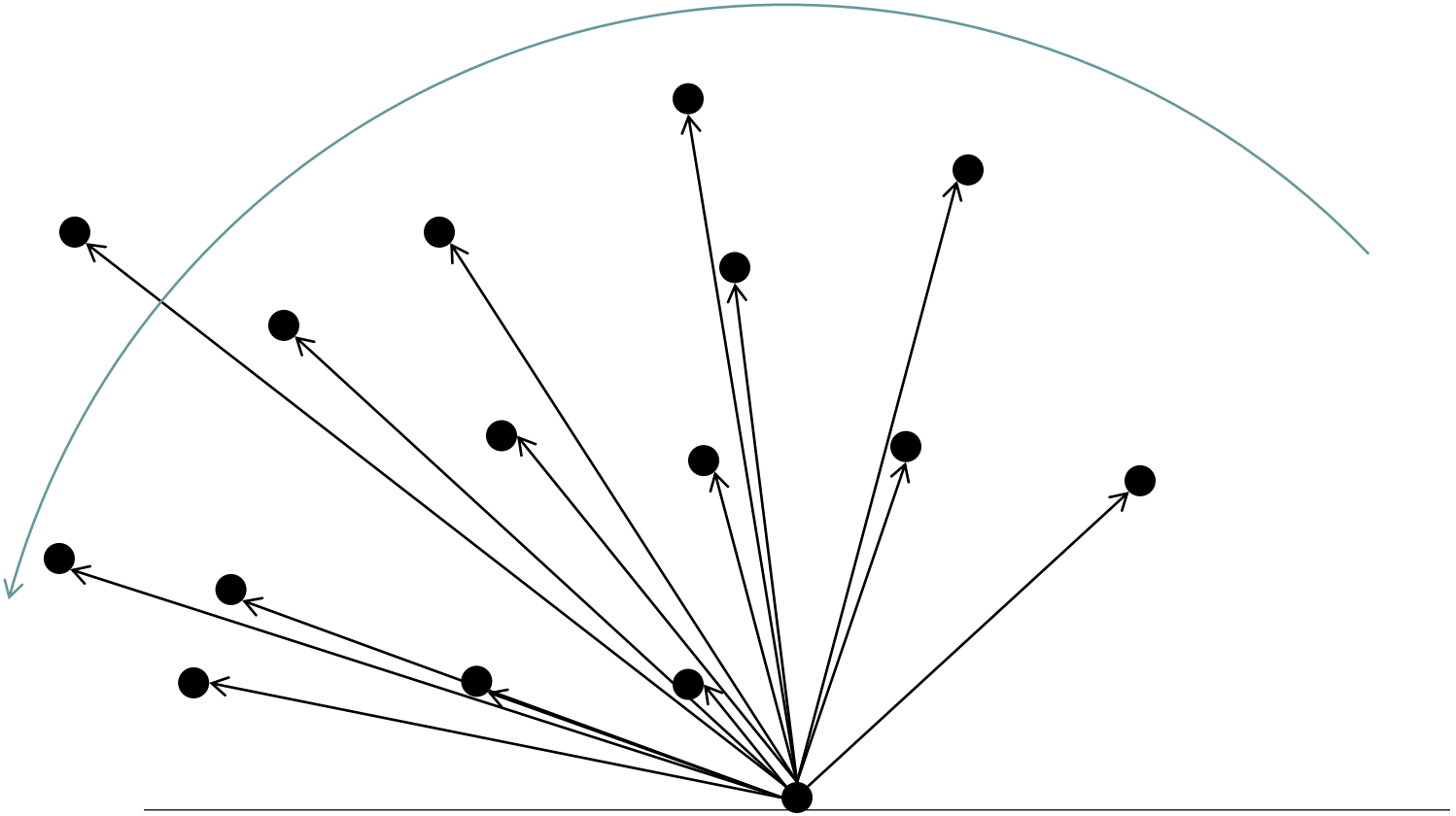
- Iterate over all possible line segments
- A line segment is part of the convex hull if all other points are to its left
- Emit all segments in a CCW order

- Running time $O(n^3)$

Naïve Convex Hull Algorithm



Graham Scan Algorithm



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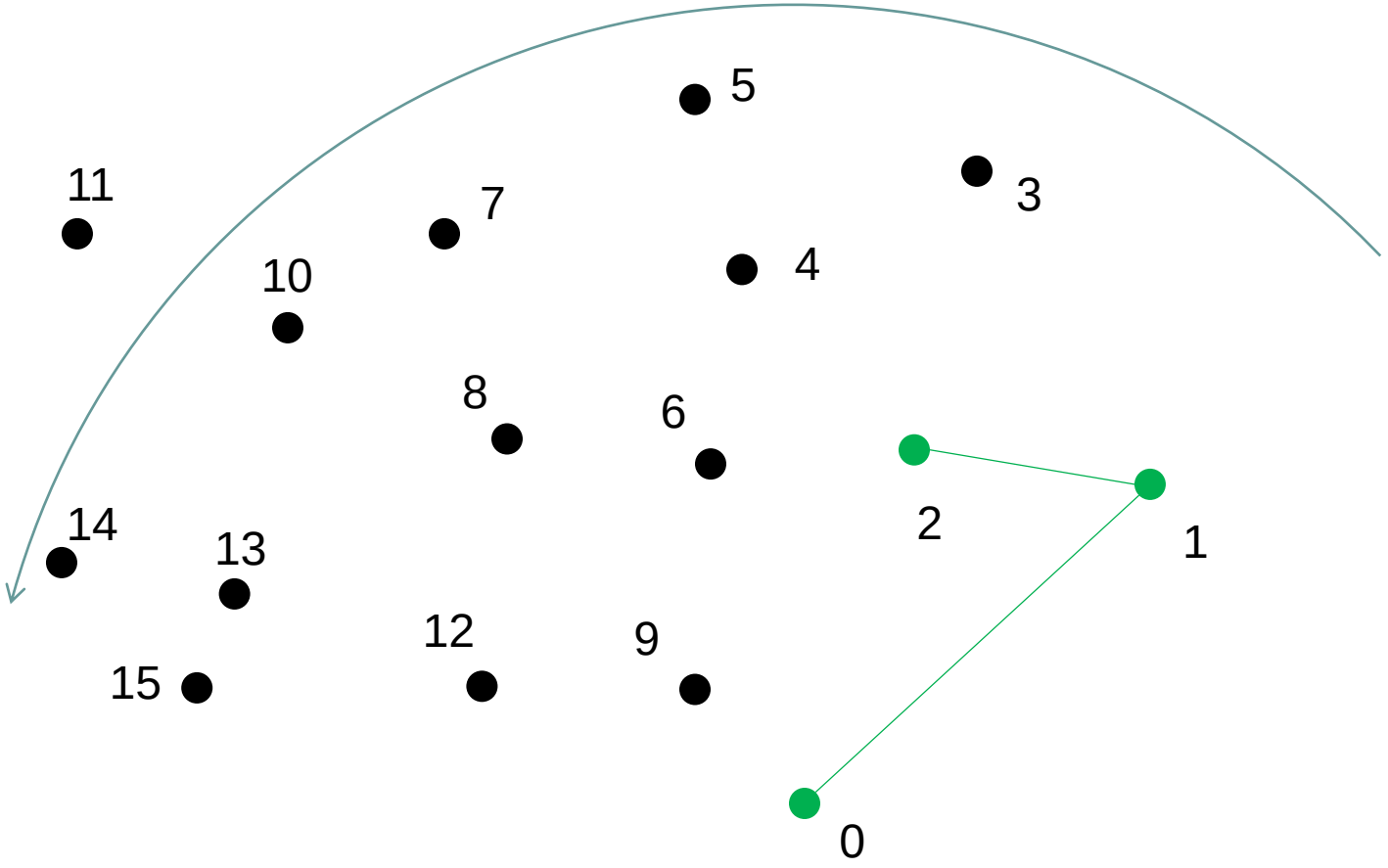

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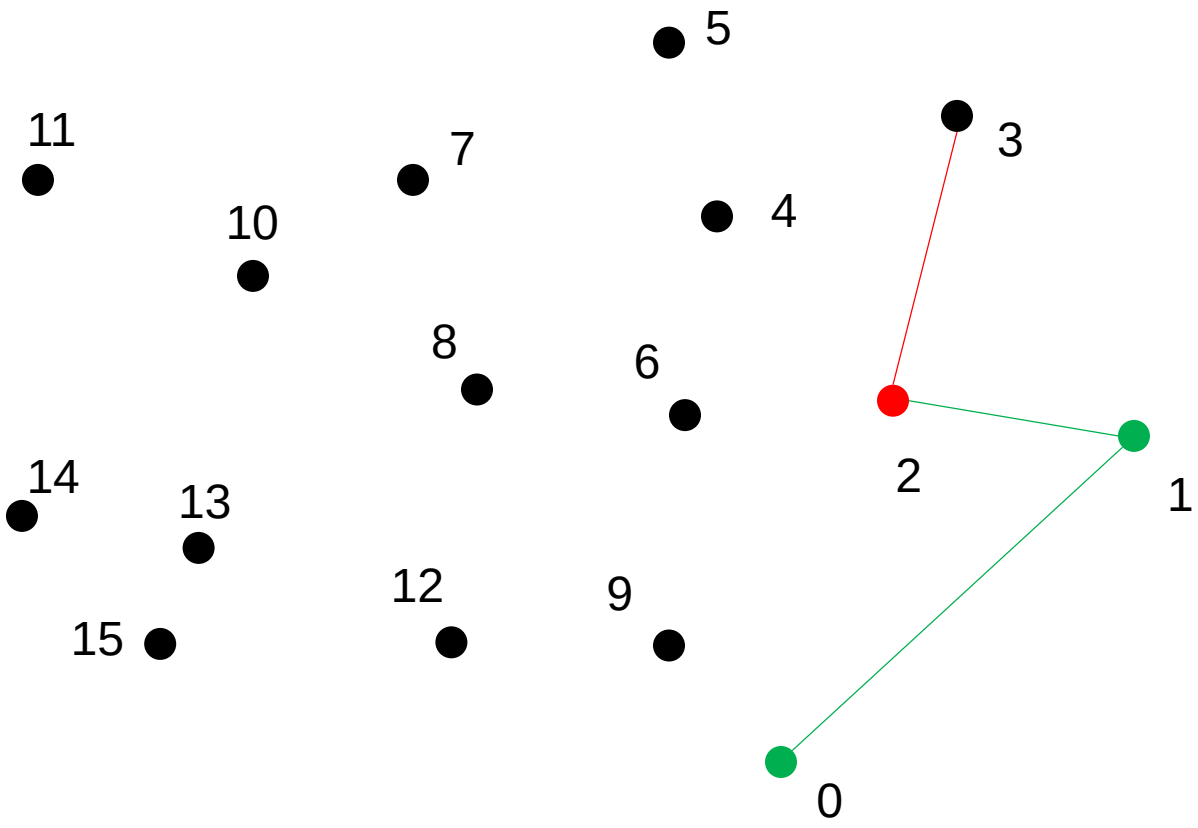

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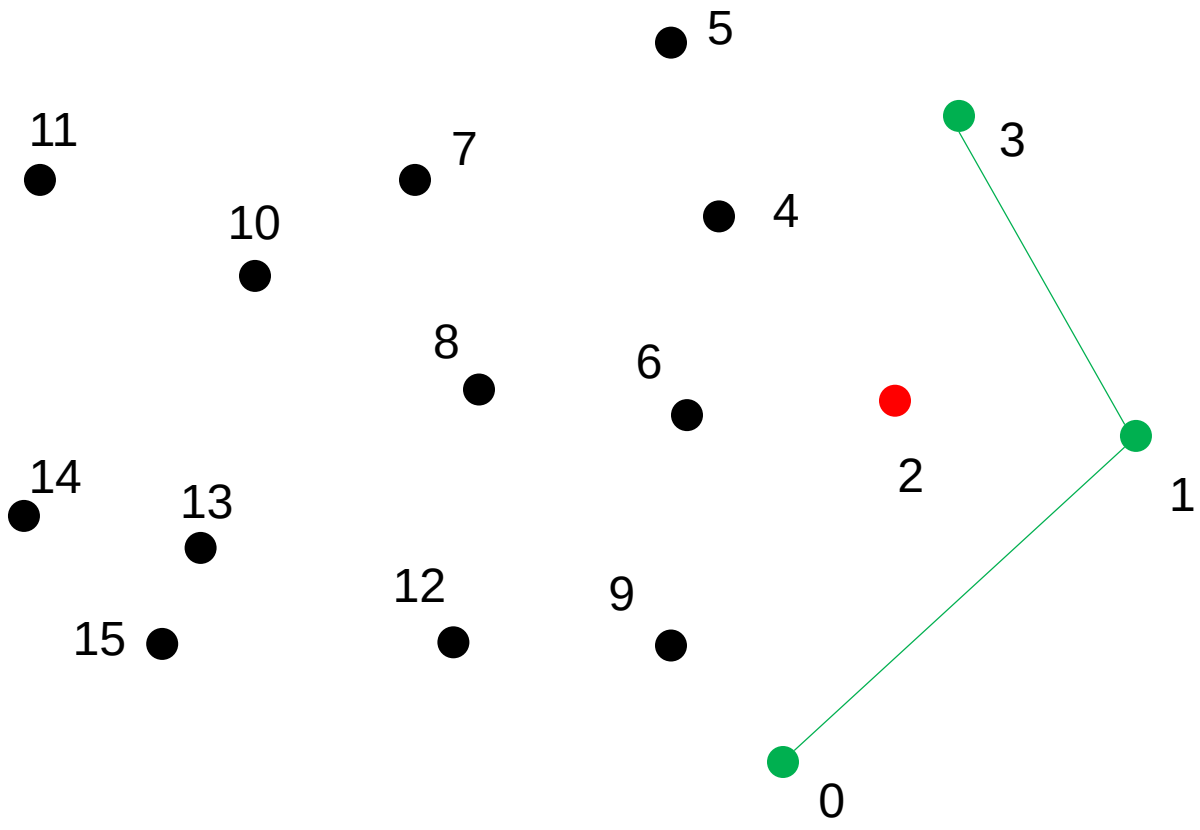

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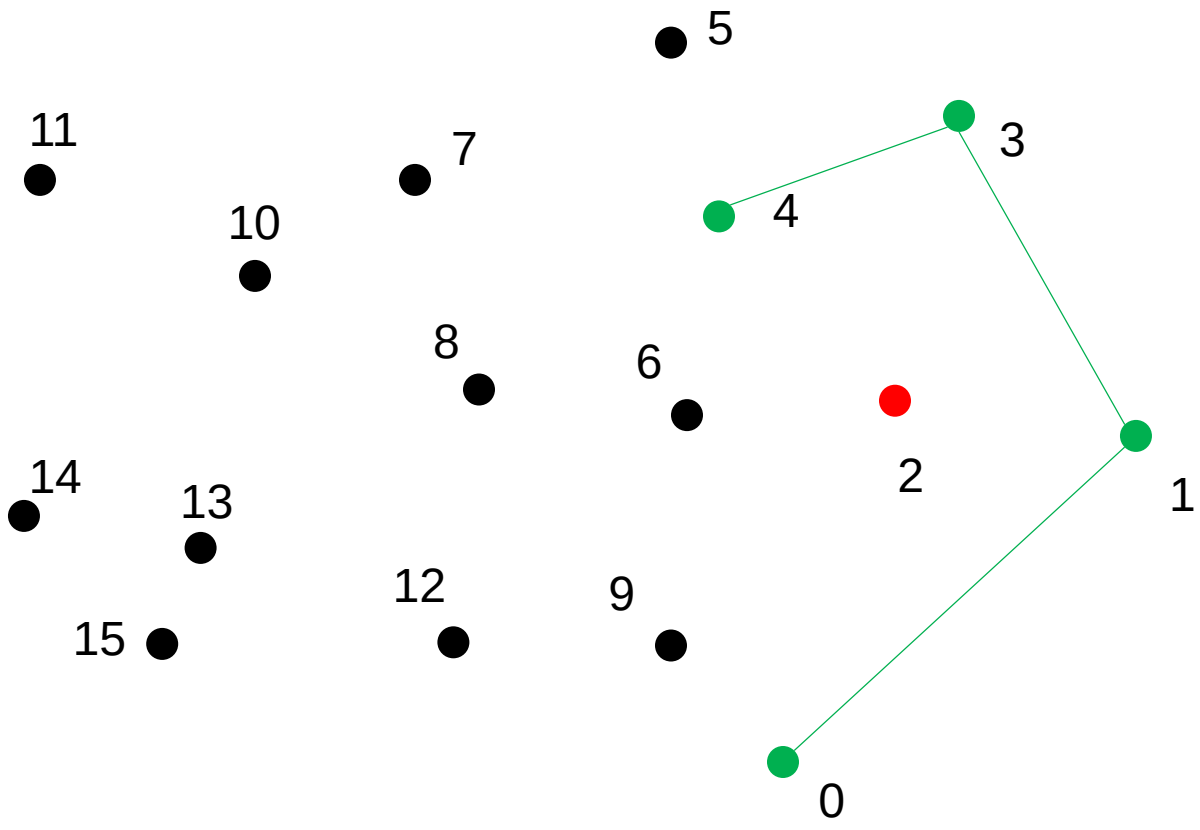

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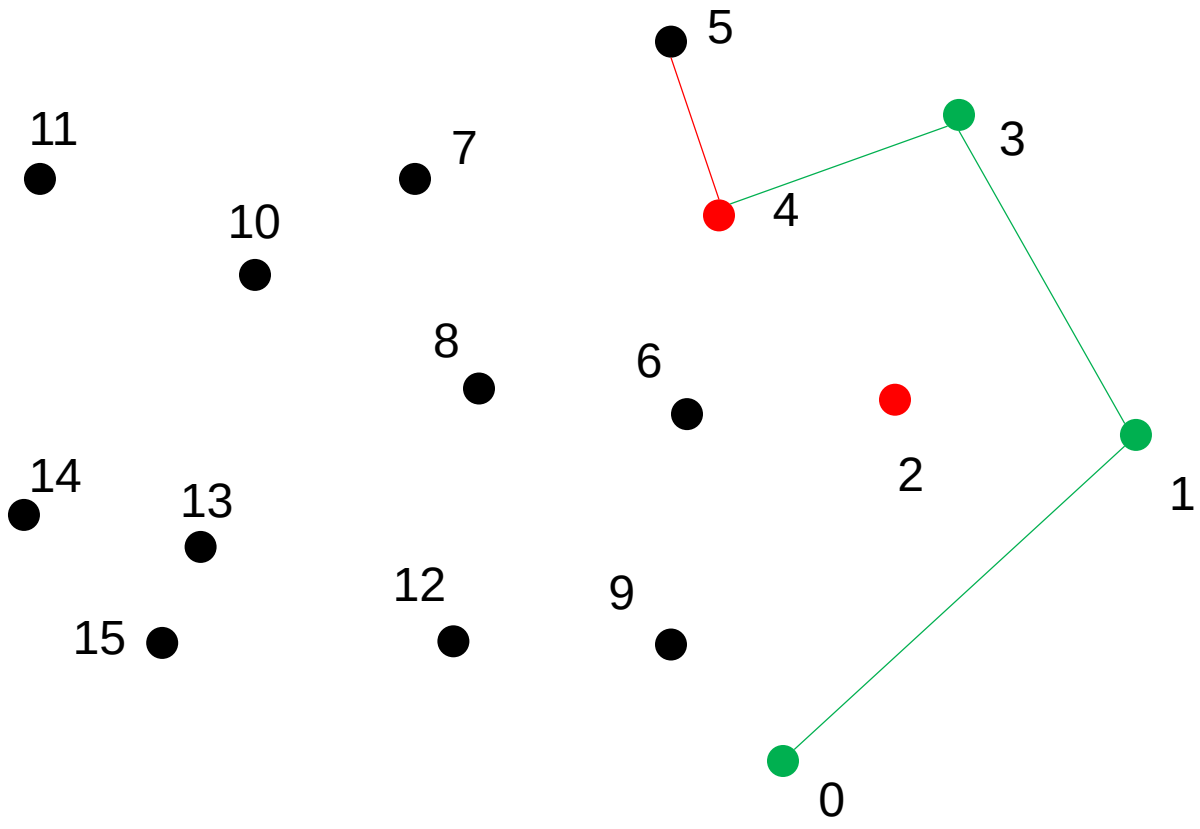

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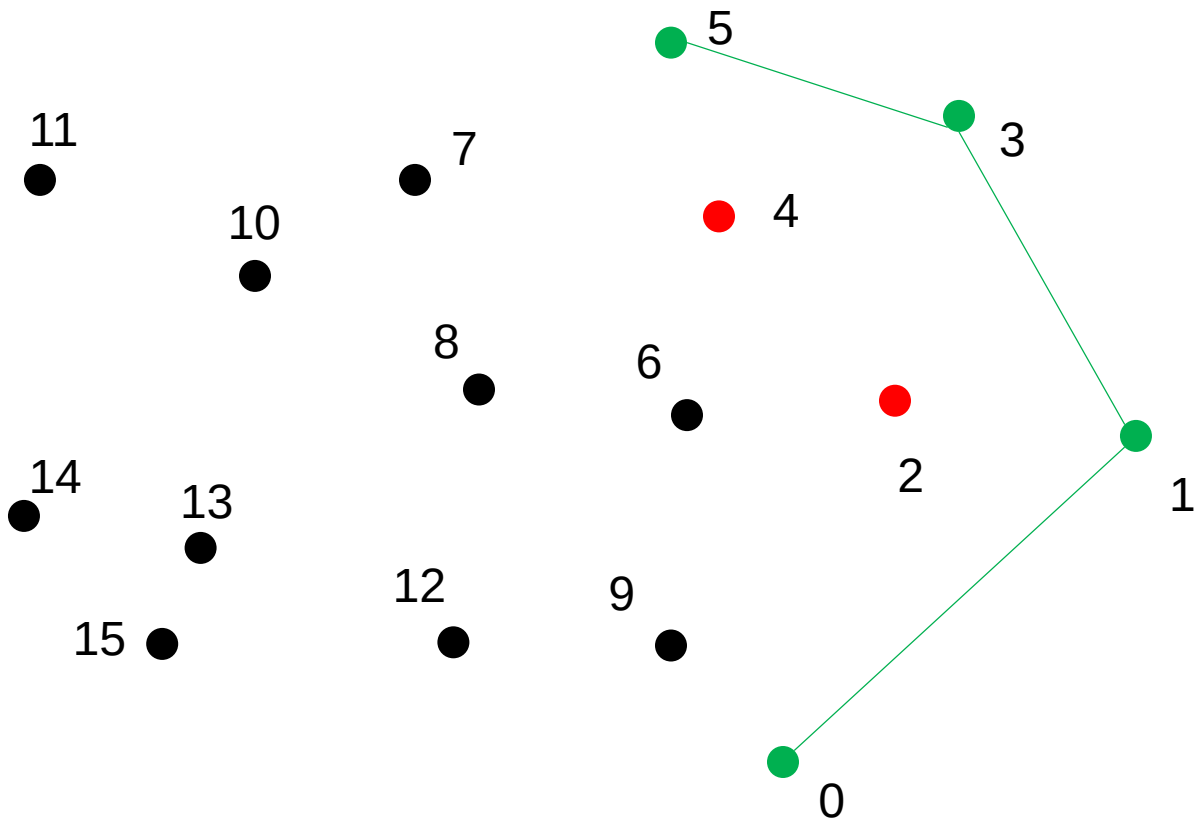

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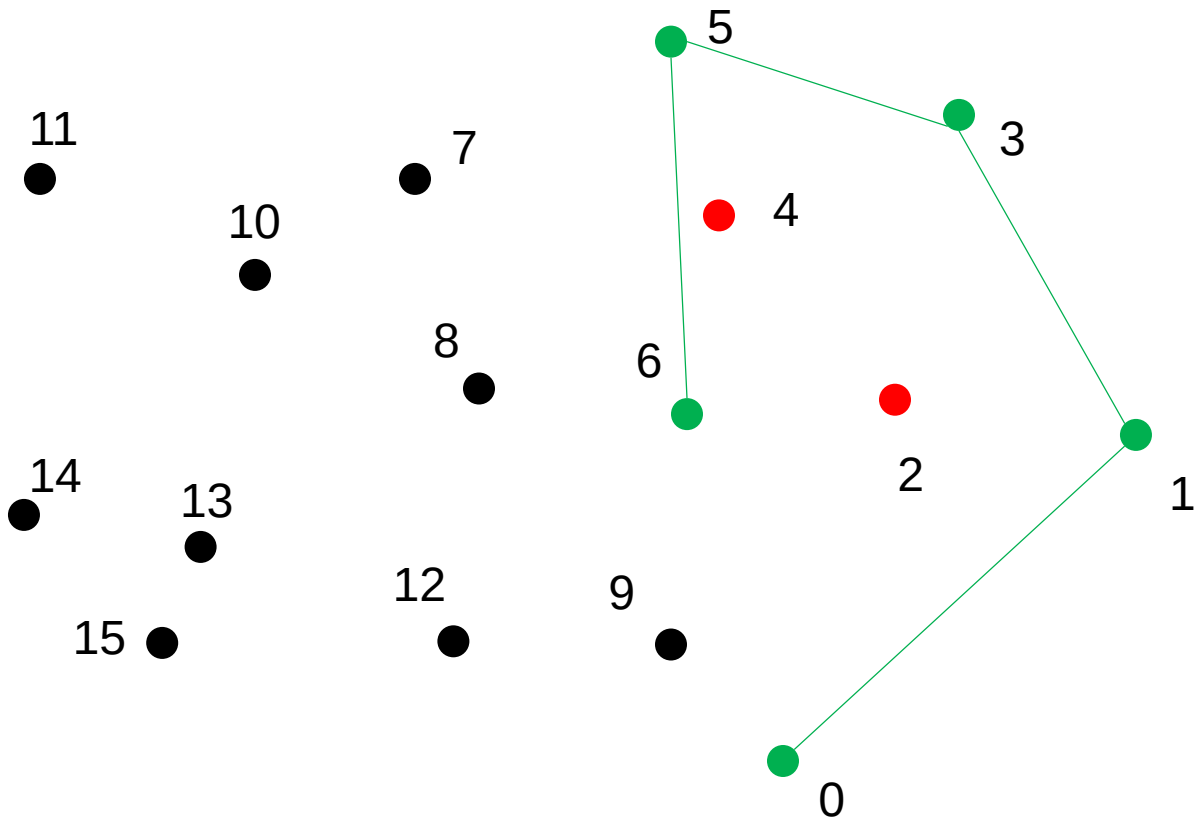

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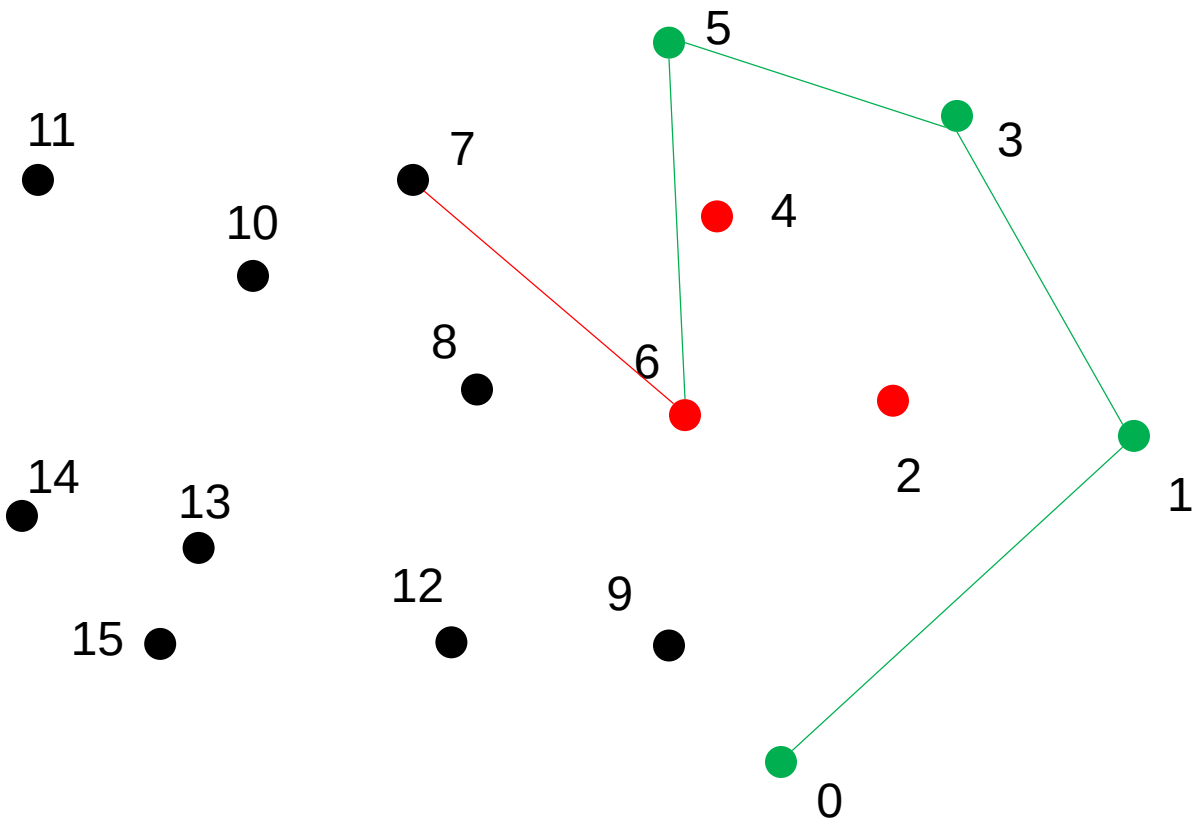

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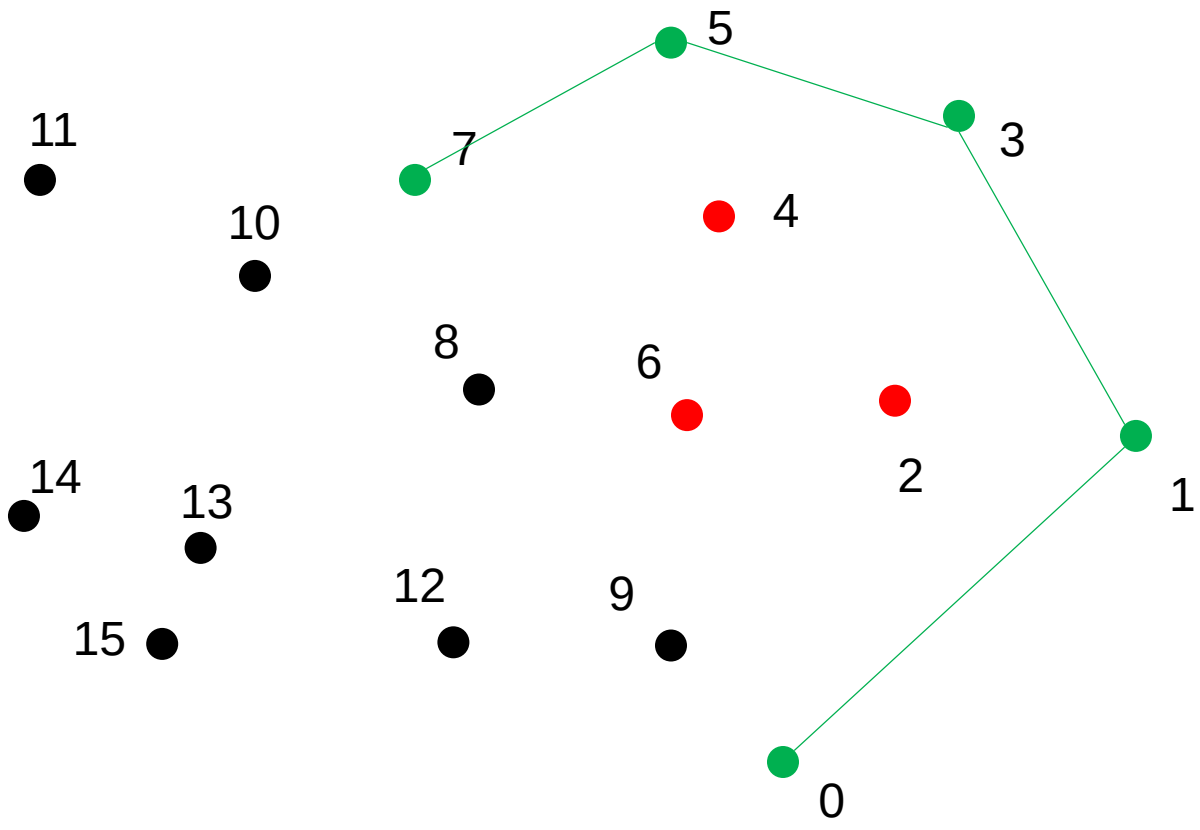

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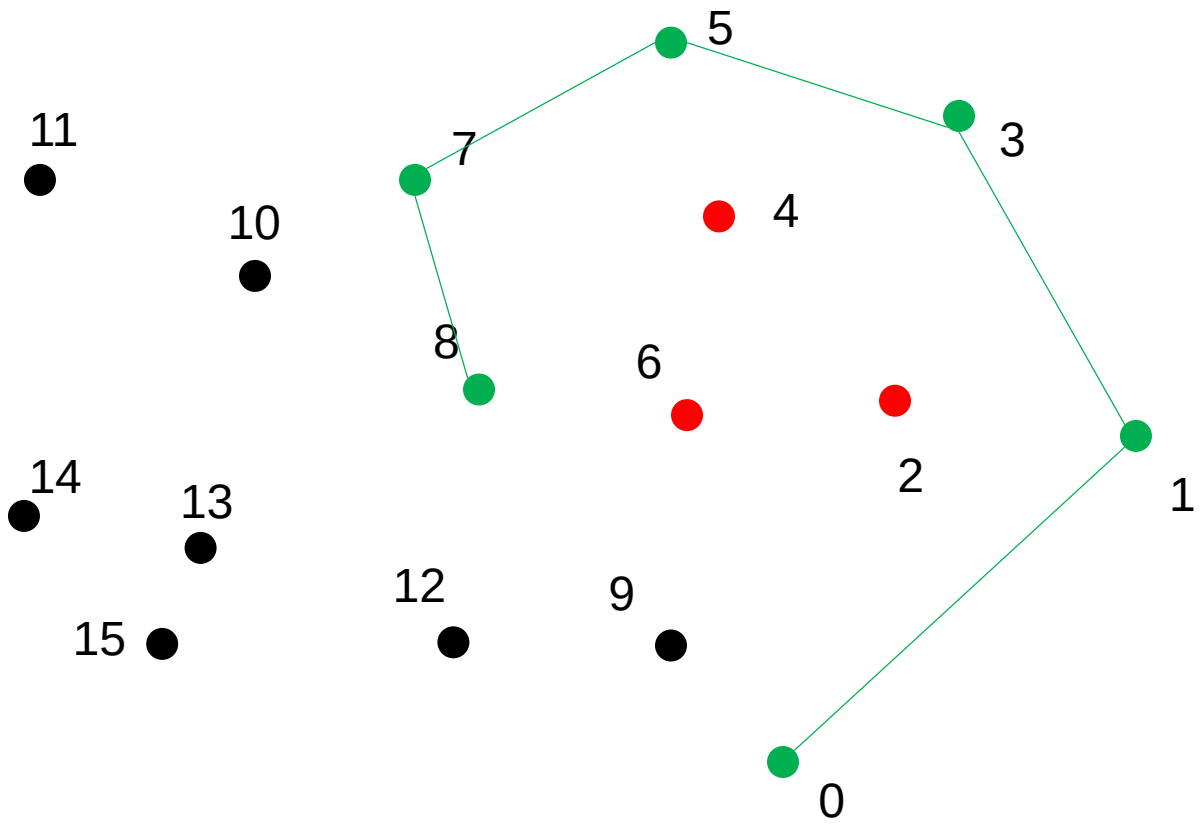

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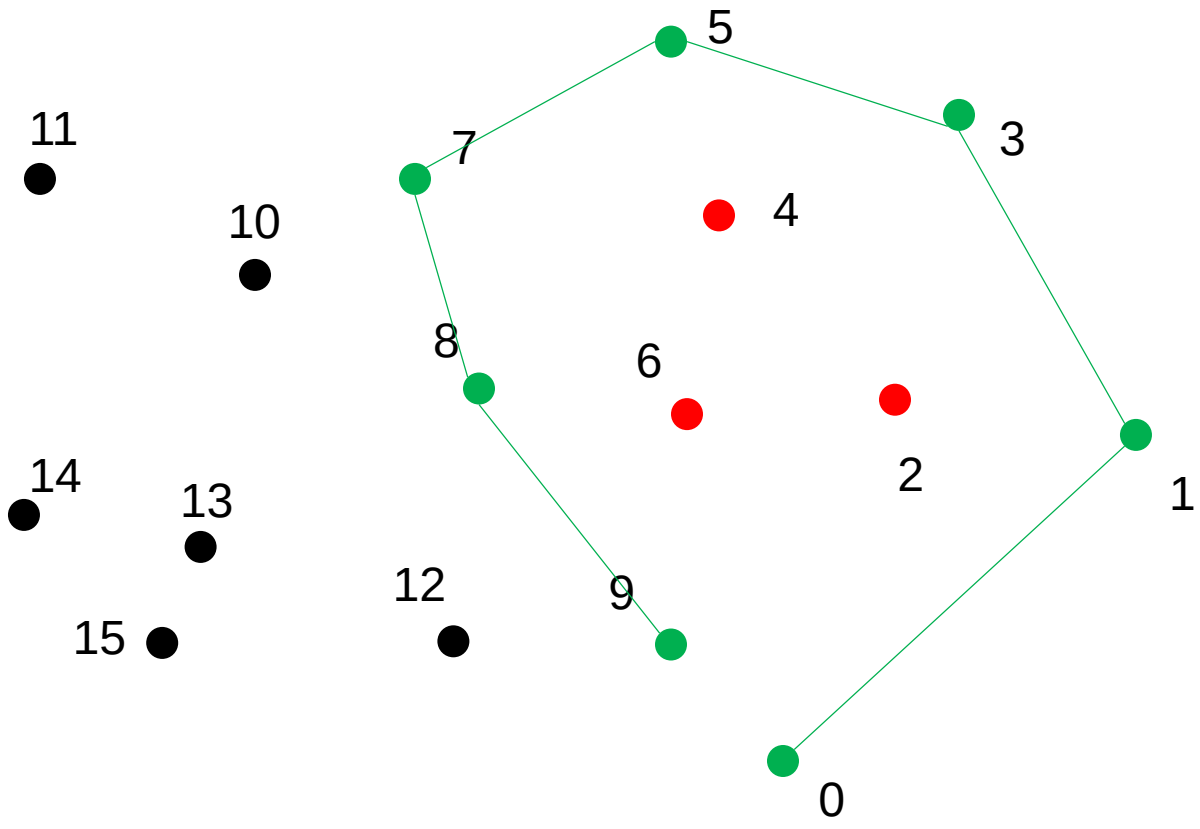

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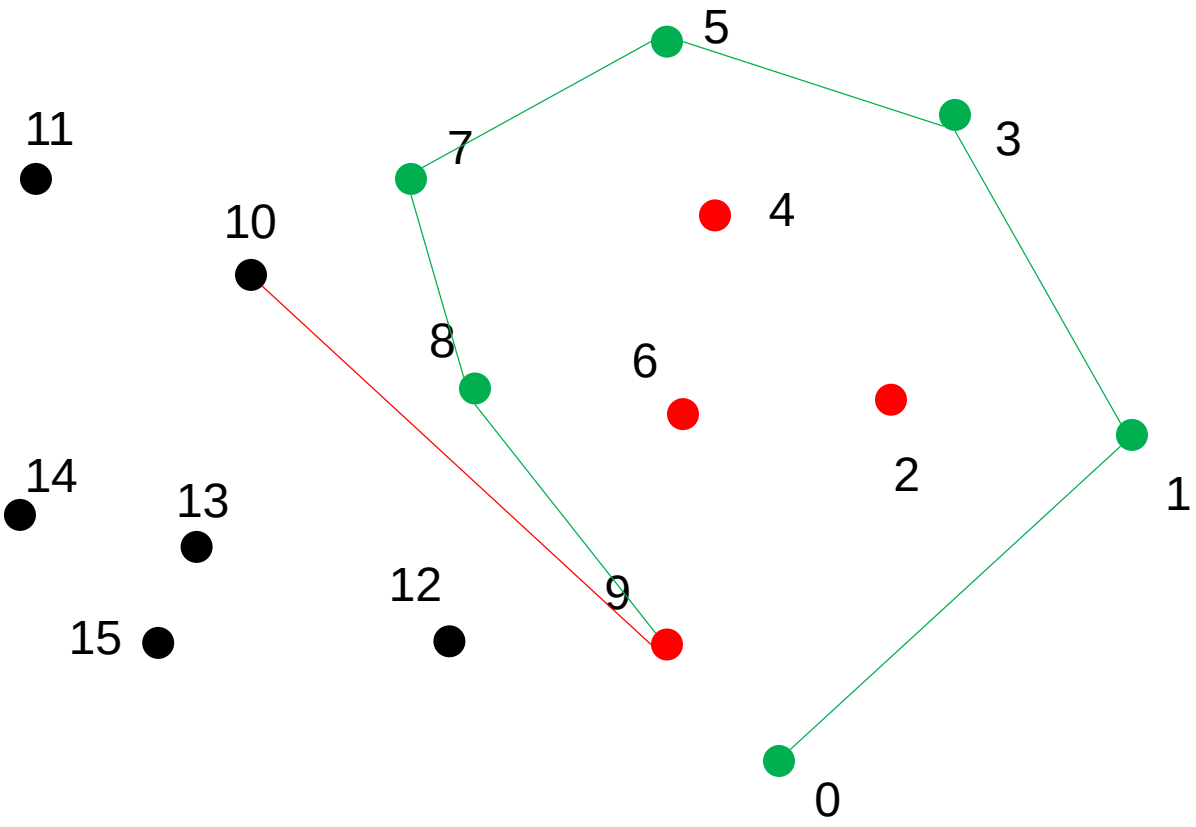

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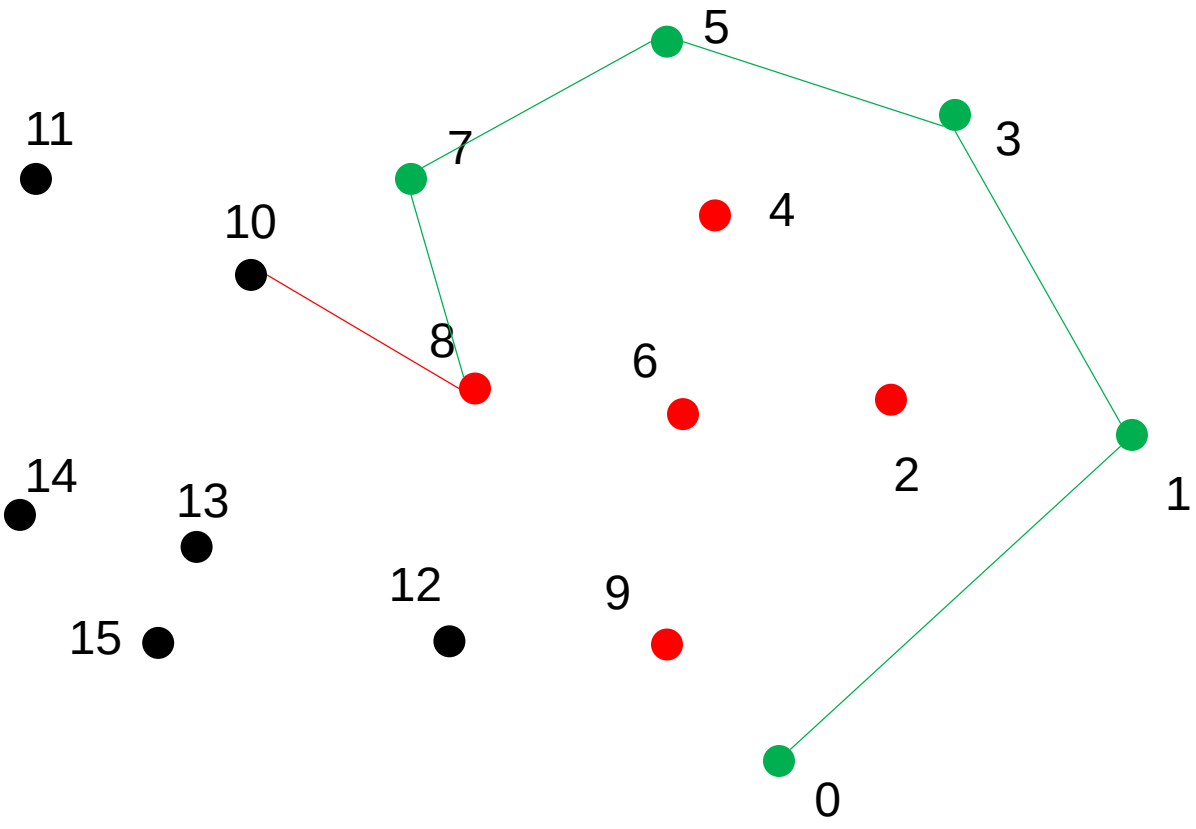

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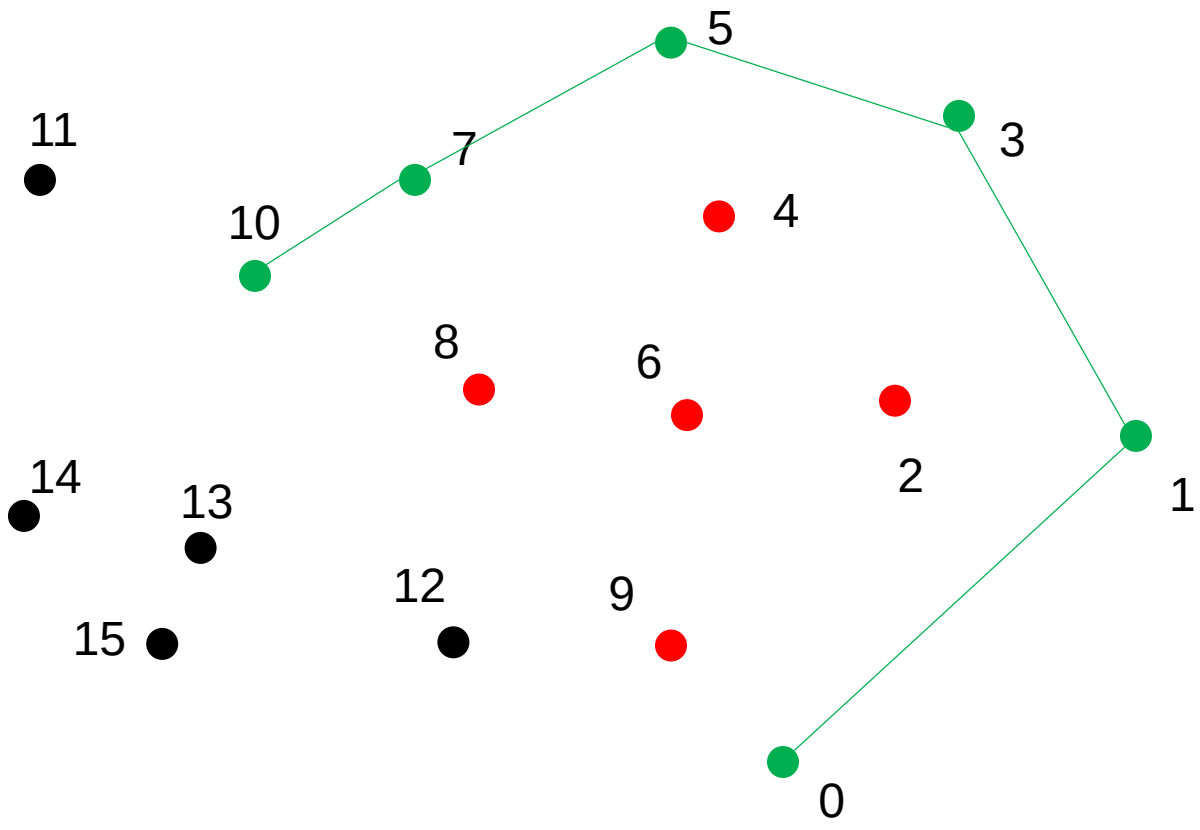

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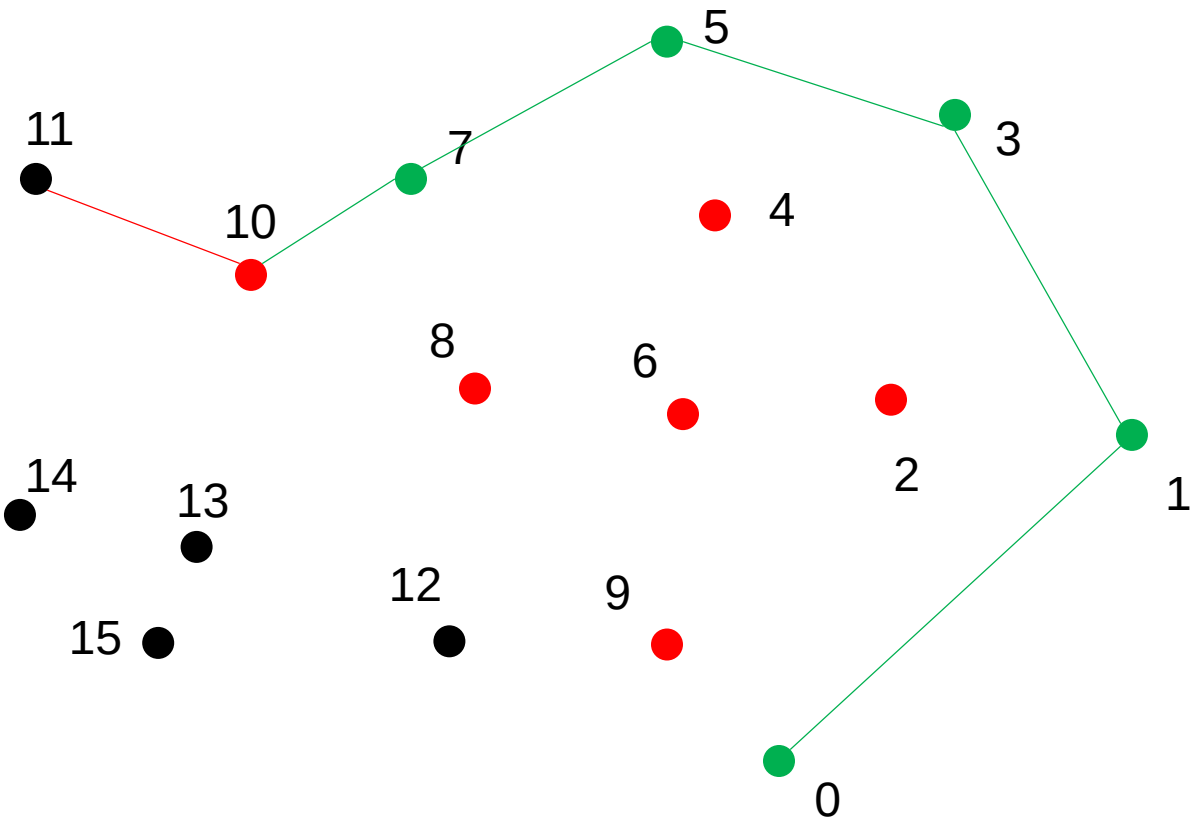

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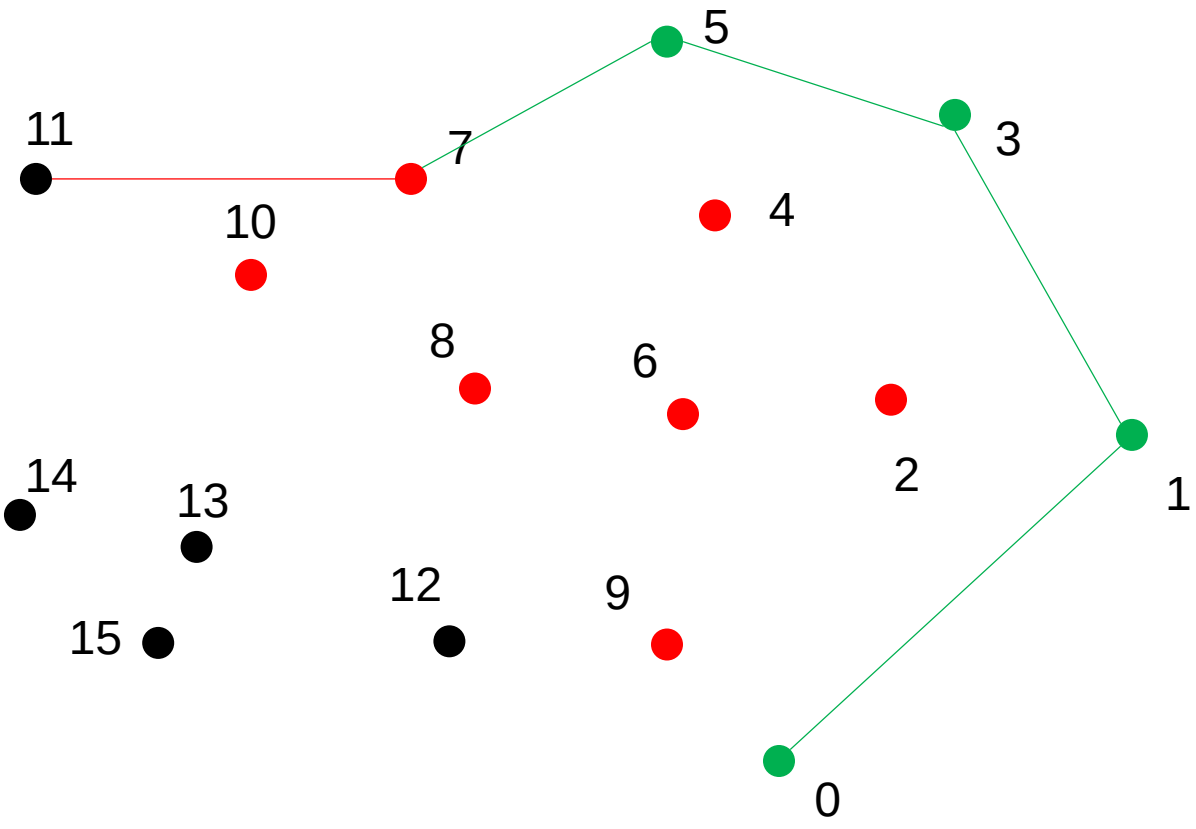

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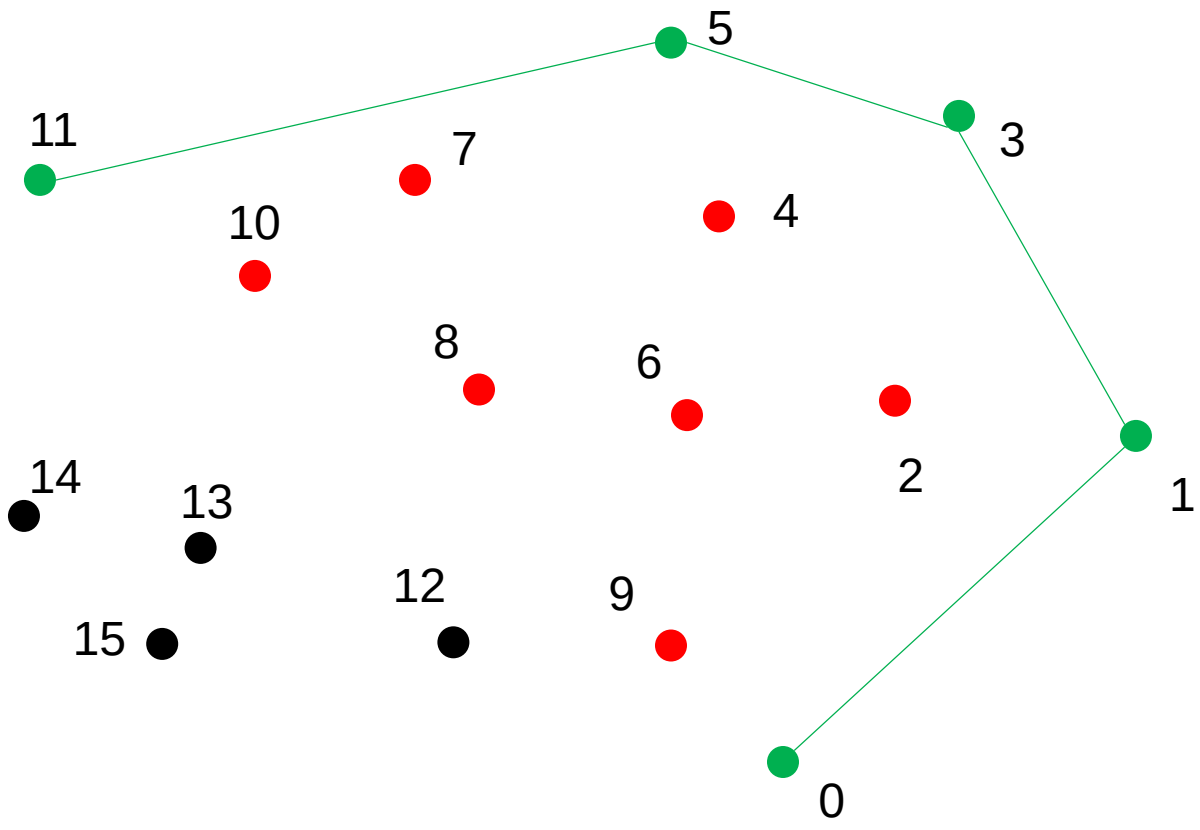

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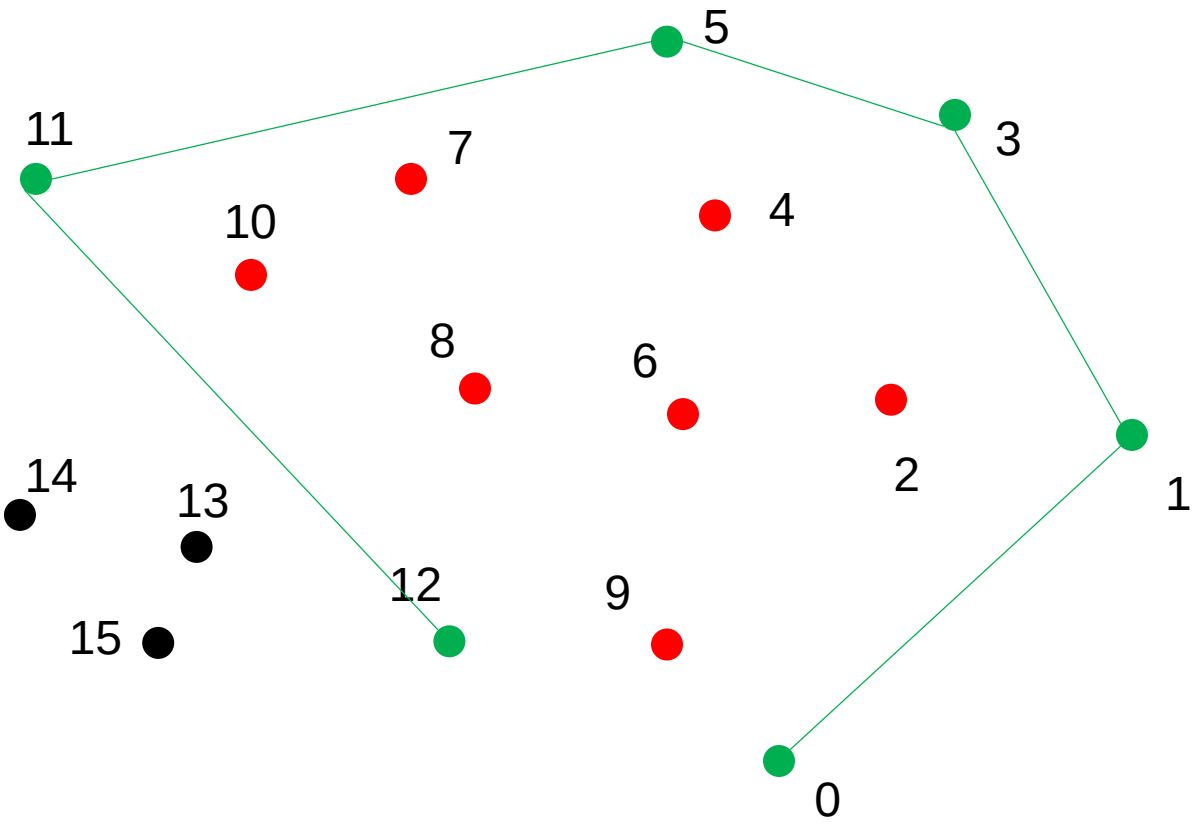

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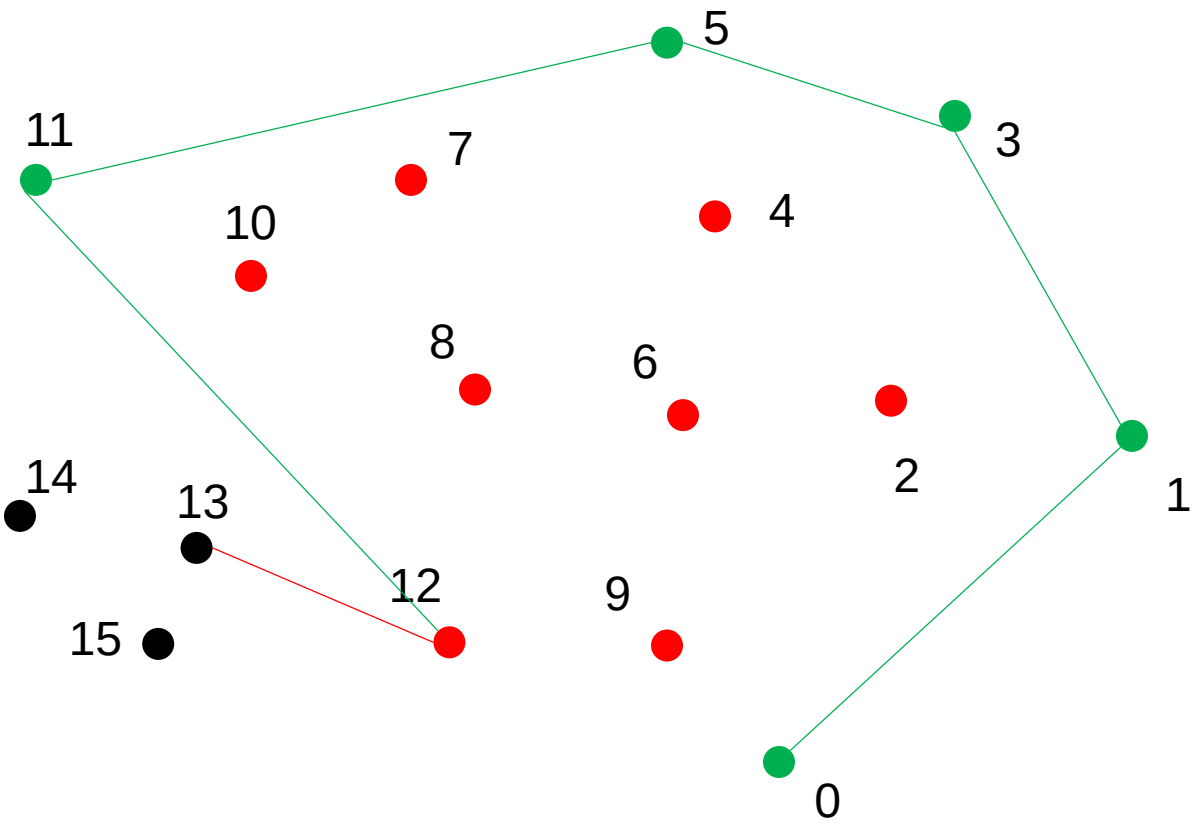

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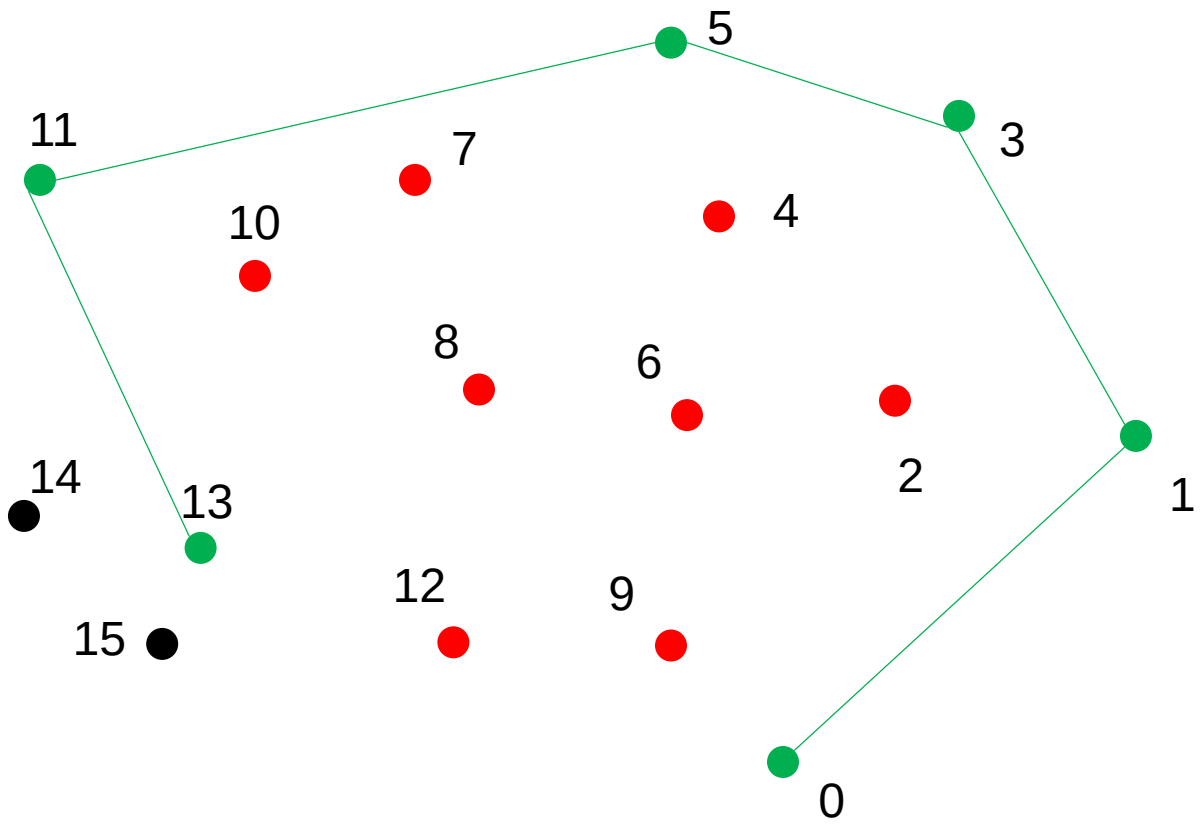

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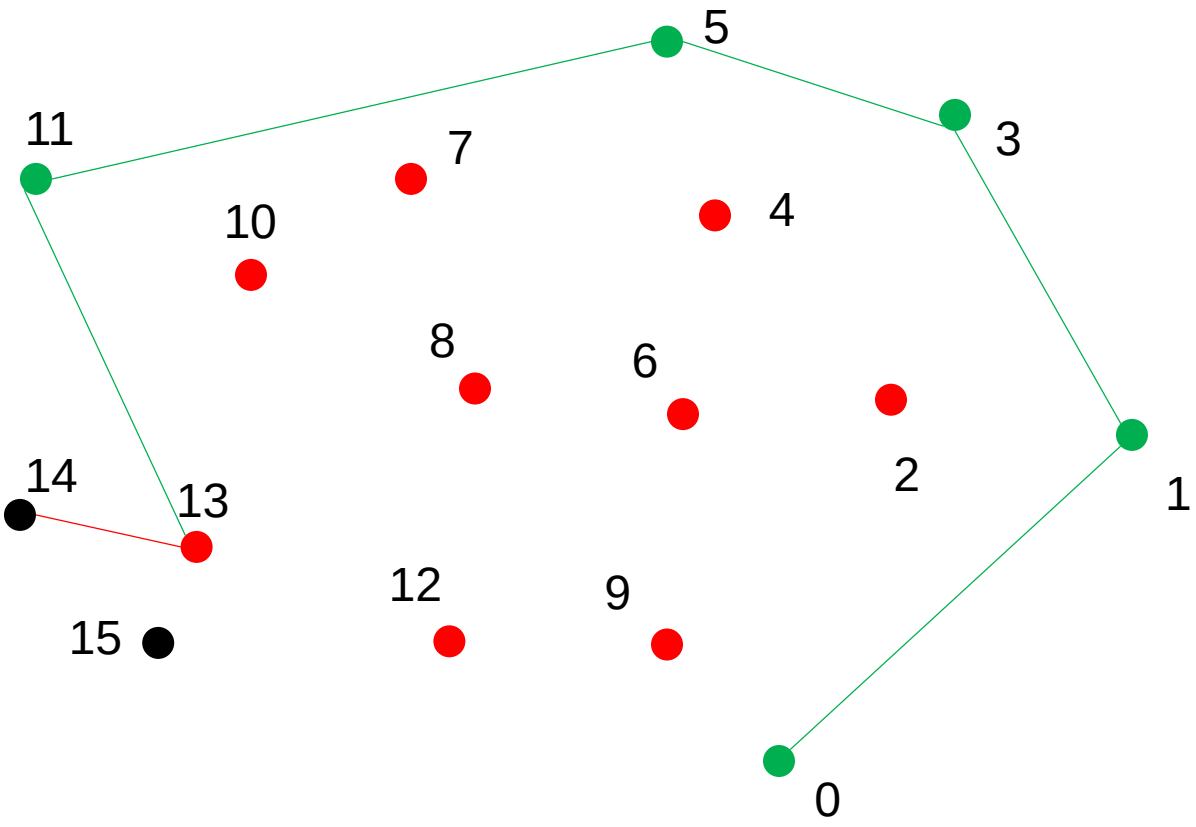

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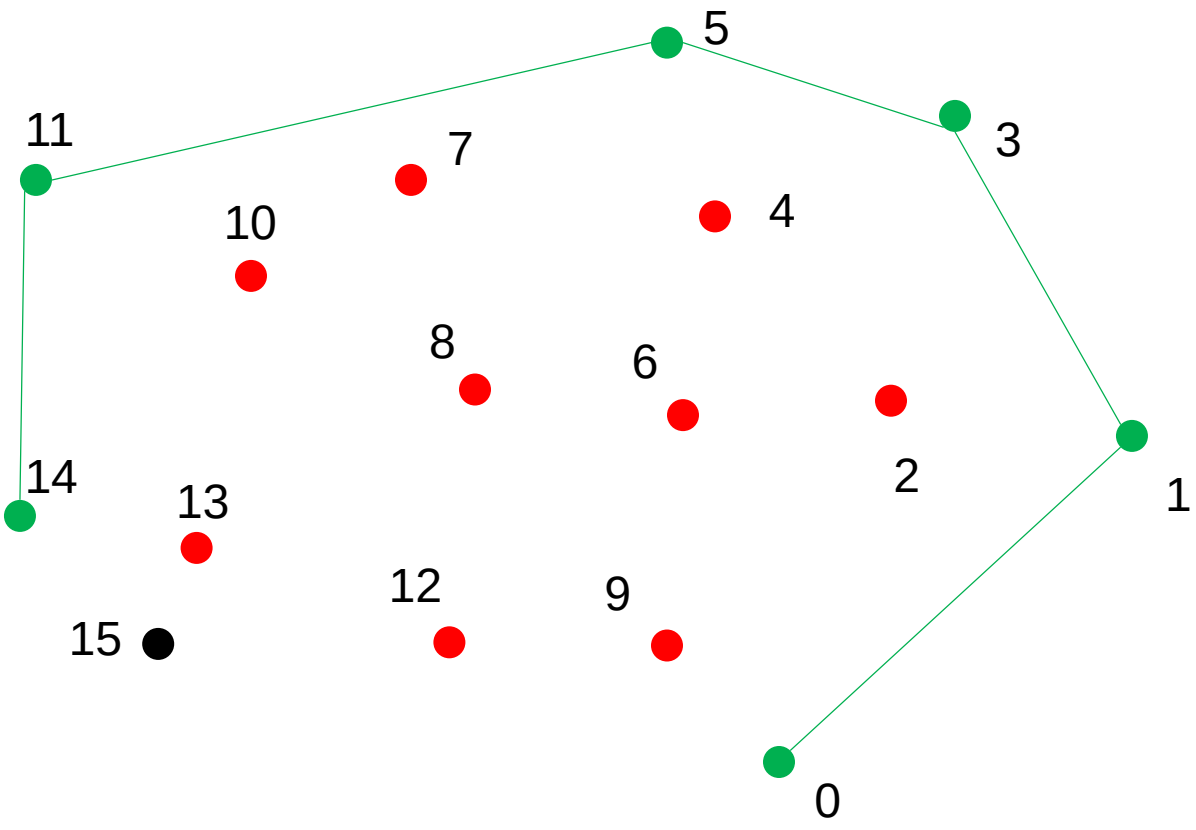

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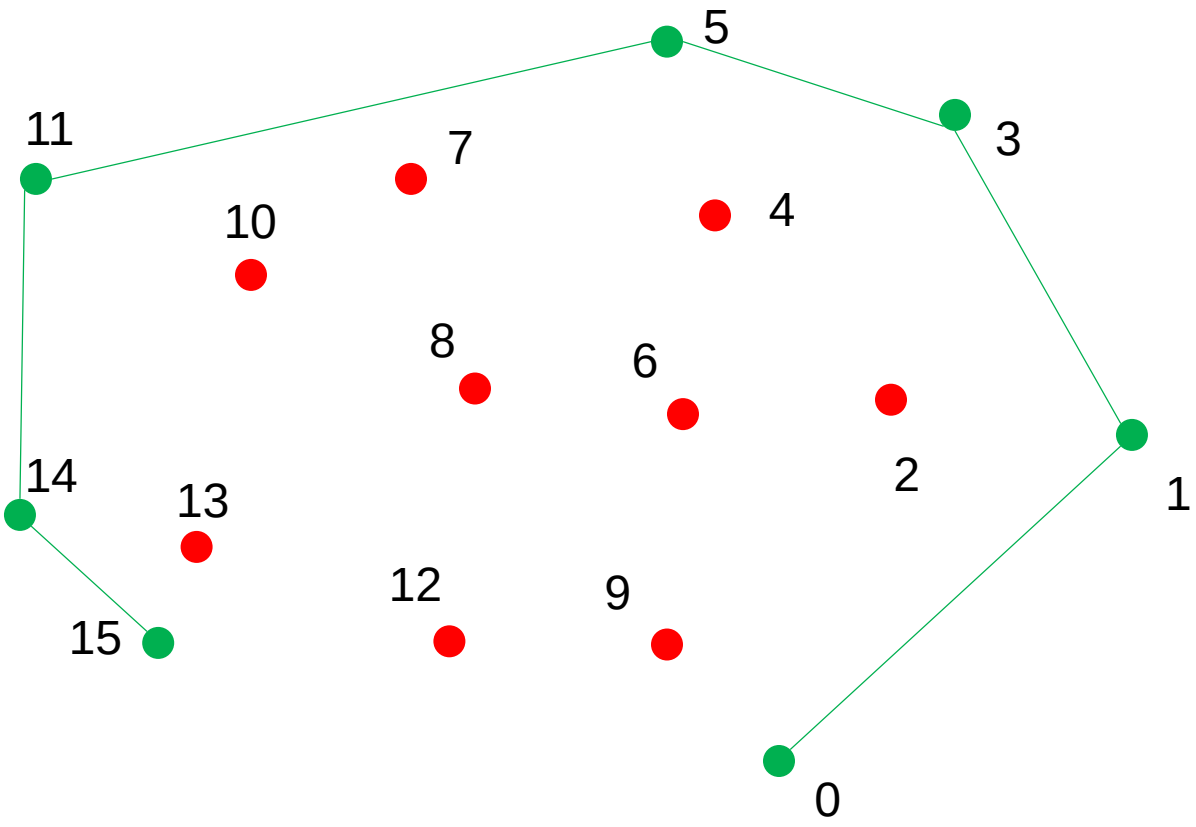

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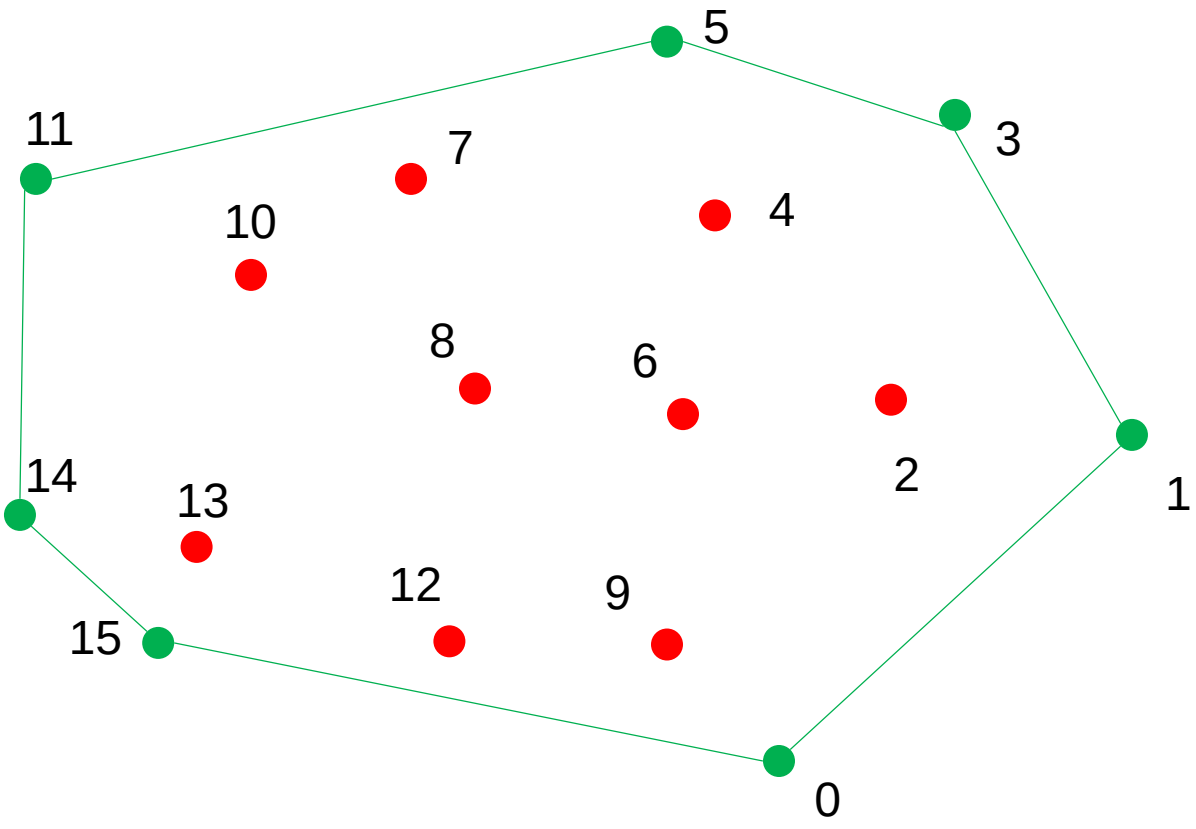

Graham Scan Pseudo Code

- Select the point with minimum y
- Sort all points in CCW order $\{p_0, p_1, \ldots, p_n\}$
- $S = \{p_0, p_1\}$
- For $i = 2$ to n
  - While $|S| > 2$ && p_i is to the right of S_{-2}, S_{-1}
    - S.pop
  - S.push(p_i)