Προχωρημένη Κατανεμημένη Υπολογιστική

HY623

Διδάσκων –
Δημήτριος Κατσαρός

@ Τμ. ΗΜΜΥ
Πανεπιστήμιο Θεσσαλίας

Διάλεξη 6η

Bloom Filter

Approximate membership queries

Lookup problem

- Given a set $S = \{x_1, x_2, x_3, \ldots x_n\}$ on a universe U, want to answer queries of the form:

$$\boxed{is\ \ y \in S\,?}$$

- Example: a set of URLs from the universe of all possible URL strings
- Bloom Filter provides an answer in
  - "Constant" time (time to hash)
  - Small amount of space
  - But with some probability of being wrong

Bloom Filters

Start with an m bit array, filled with 0s.

B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Hash each item x_j in S k times. If $H_i(x_j) = a$, set $B[a] = 1$.

B	0	1	0	0	1	0	1	0	0	1	1	1	0	1	1	0

To check if y is in S, check B at $H_i(y)$. All k values must be 1.

B	0	1	0	0	1	0	1	0	0	1	1	1	0	1	1	0

Possible to have a false positive; all k values are 1, but y is not in S.

B	0	1	0	0	1	0	1	0	0	1	1	1	0	1	1	0

(A toy) Example

- Number of elements n= 2: 9 and 11
- Size of Bloom Filter m=5
- Number of hash functions k=2
  - $h_1(x) = x \bmod 5$
  - $h_2(x) = (2x+3) \bmod 5$

	$h_1(x)$	$h_2(x)$
Initialize		
insert 9	4	1
insert 11	1	0

Bloom Filter

0	0	0	0	0
0	1	0	0	1
1	1	0	0	1

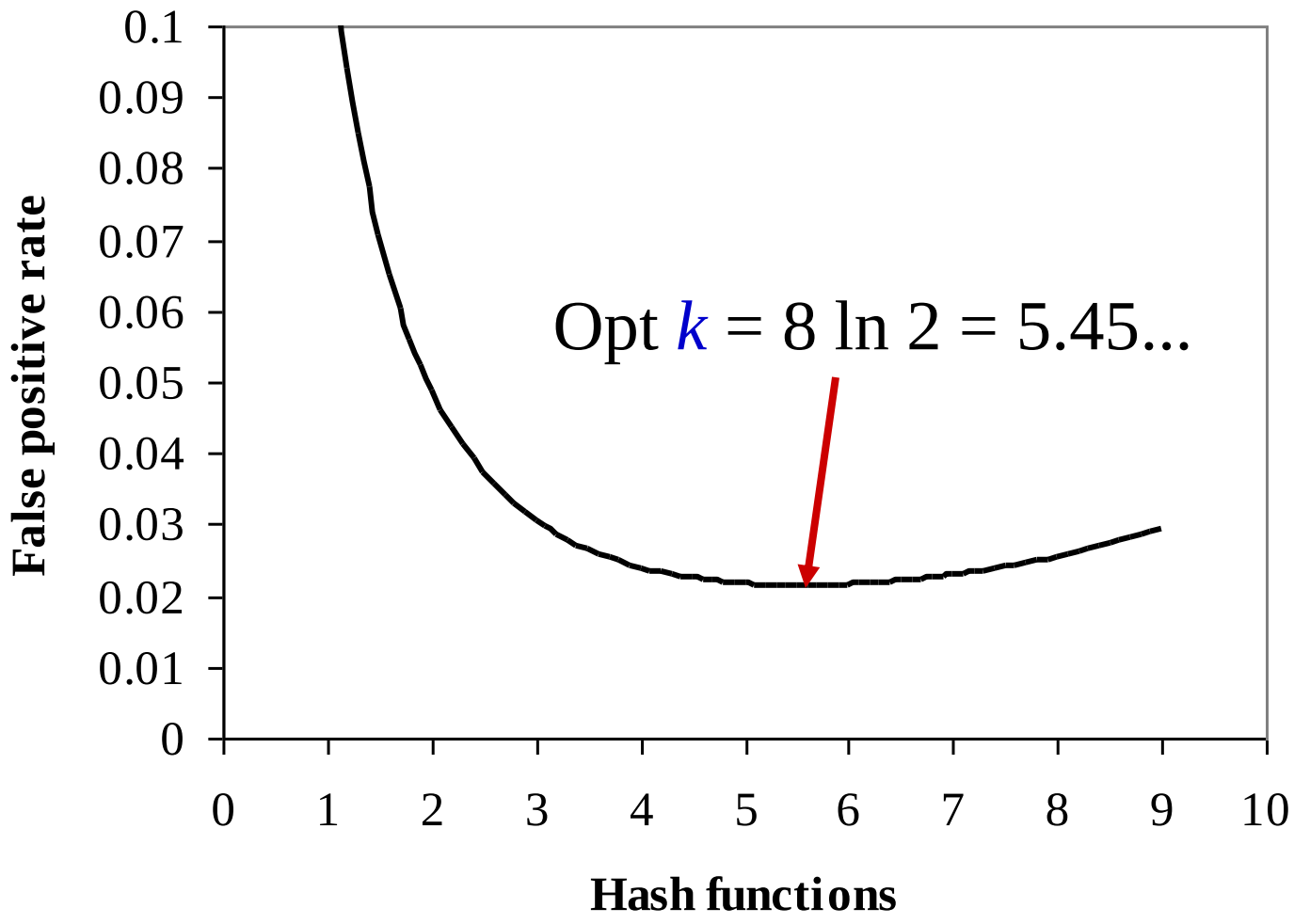
<u>Membership queries</u>

Queries	$h_1(x)$	$h_2(x)$	Answer
for elem 15	0	3	No, not in Bloom Filter (correct answer)
for elem 16	1	0	Yes, in B (wrong answer: false positive)

Errors

- Assumption: We have good hash functions, look random.
- Given m bits for filter and n elements, choose number k of hash functions to minimize false positives:
  - Let $p = \Pr[\text{cell is empty}] = (1 - 1/m)^{kn} \approx e^{-kn/m}$
  - Let $f = \Pr[\text{false pos}] = (1 - p)^k \approx (1 - e^{-kn/m})^k$
- As k increases, more chances to find a 0, but more 1's in the array.
- Find optimal at $k = (\ln 2)m/n$ by calculus (scanned document accompanying this lecture)

Example


0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
False positive rate
0
1
2
3
4
5
6
7
8
9
10
Hash functions
Opt k = 8 ln 2 = 5.45...

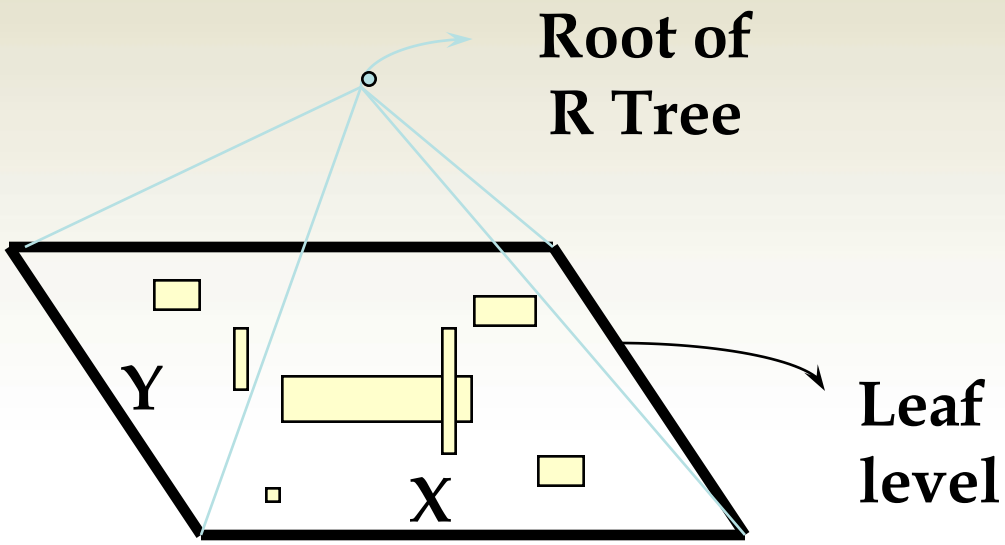
$m/n = 8$

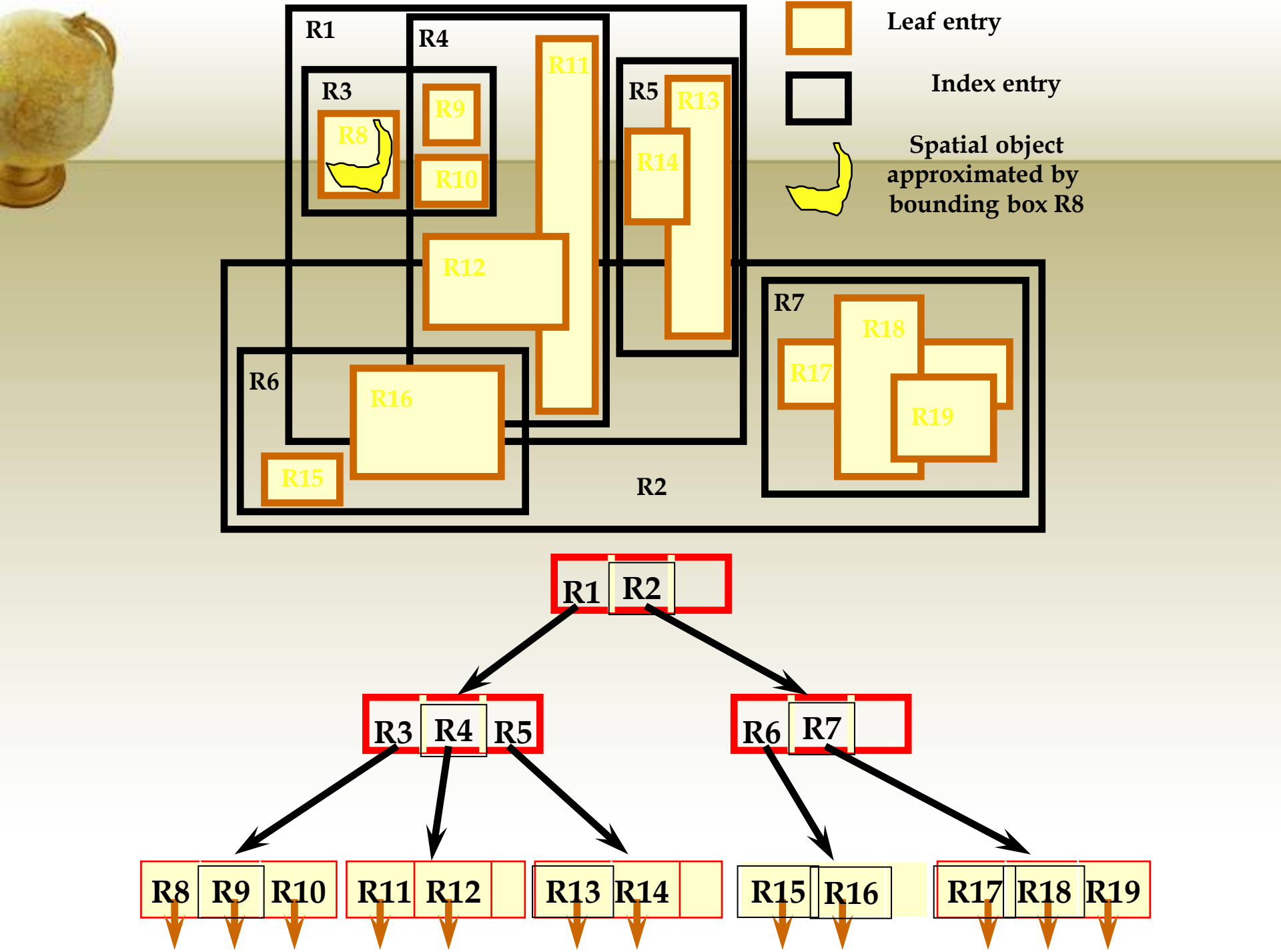
R-tree

Indexing multi-dimensional data

The R-Tree

- The R-tree is a tree-structured index that remains balanced on inserts and deletes.
- Each key stored in a leaf entry is intuitively a box, or collection of intervals, with one interval per dimension.
- Example in 2-D:




R1
R4
R3
R8
R9
R10
R11
R5
R13
R14
R12
R7
R18
R17
R19
R6
R16
R15
R2
Leaf entry
Index entry
Spatial object approximated by bounding box R8
R1 R2
R3 R4 R5
R6 R7
R8 R9 R10
R11 R12
R13 R14
R15 R16
R17 R18 R19