# Nєиро-Абаци́ऽ Үлодоүıбтьки́ Neuro-Fuzzy Computing 

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## Generalization for MLP: Dropout

## Bagging: Bootstrap aggregating

## Ensemble

Train a bunch of networks with different structures


## Bagging: Bootstrap aggregating



## Dropout



- Each time before forward and backward phase
- Each neuron has p\% to dropout


## Dropout: Some neurons are eliminated



- The marked neurons (and their associated connections) are deleted


## Dropout: Changed topology



- The structure of the network is changed
- Using the new network for training


## Dropout: Testing



## No dropout

- If the dropout rate at training is $\mathrm{p} \%$, all the weights times (1-p)\%
- Assume that the dropout rate is $50 \%$. If a weight $\mathrm{w}=1$ by training, set $w=0.5$ for testing


## Dropout: A kind of ensemble






## Training of Dropout

M neurons

$2^{\mathrm{M}}$ possible networks

## Dropout: A kind of ensemble



## Dropout for a single linear unit

A unit computing a weighted sum of $n$ inputs of the form:

$$
S=S(I)=\sum_{i=1}^{n} w_{i} I_{i}
$$

where $I=\left(I_{1}, I_{2}, \ldots, I_{\mathrm{n}}\right)$ is the input vector.
If we delete inputs with a uniform distribution over all possible subsets of inputs, or equivalently with a probability $q=0.5$ of deletion, then there are $2^{\mathrm{n}}$ possible networks, including the empty network. For a fixed $I$, the average output over all these networks can be written as:

$$
E(S)=\frac{1}{2^{n}} \sum_{\mathcal{N}} S(\mathcal{N}, I)
$$

where $N$ is used to index all possible sub-networks, i.e., all possible edge deletions. In this simple case, deletion of input units or of edges are the same thing.

## Dropout for a single linear unit

This sum can be expanded using networks of size $0,1,2, \ldots, n$ in the form:


In this expansion, the term $w_{\mathrm{i}} I_{\mathrm{i}}$ occurs

$$
1+\binom{n-1}{1}+\binom{n-2}{2}+\cdots+\binom{n-1}{n-1}=2^{n-1}
$$

times. So, finally the average output is:

$$
E(S)=\frac{2^{n-1}}{2^{n}}\left(\sum_{i=1}^{n} w_{i} I_{i}\right)=\sum_{i=1}^{n} \frac{w_{i}}{2} I_{i}
$$

Thus in the case of a single linear unit, for any fixed input I the output obtained by halving all the weights is equal to the arithmetic mean of the outputs produced by all the possible sub-networks. This combinatorial approach can be applied to other cases (e.g., p != 0.5)

