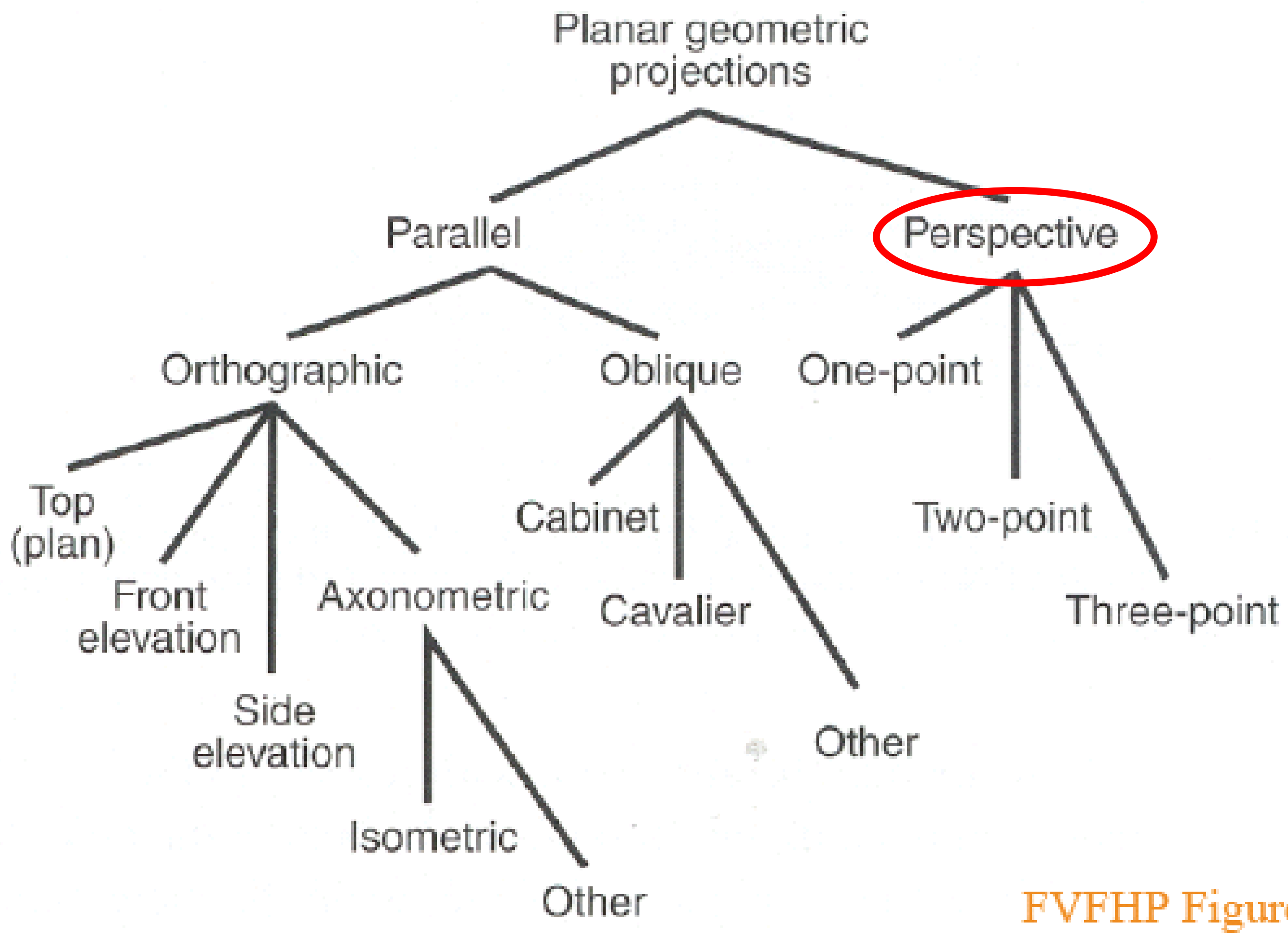
ΗΥ416
ΓΡΑΦΙΚΑ ΥΠΟΛΟΓΙΣΤΩΝ

Προβολές

Π. ΤΣΟΜΠΑΝΟΠΟΥΛΟΥ
ΠΑΝΕΠΙΣΤΗΜΙΟ ΘΕΣΣΑΛΙΑΣ
ΤΜΗΜΑ ΗΛΕΚΤΡΟΛΟΓΩΝ ΜΗΧΑΝΙΚΩΝ & ΜΗΧΑΝΙΚΩΝ ΥΠΟΛΟΓΙΣΤΩΝ

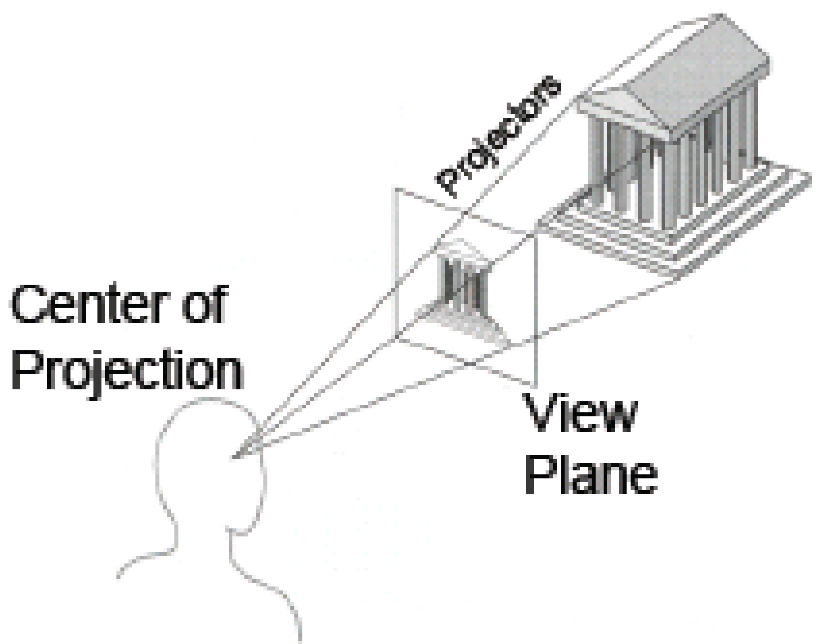
Taxonomy of Projections



FVFHP Figure 6.10

Perspective Projection

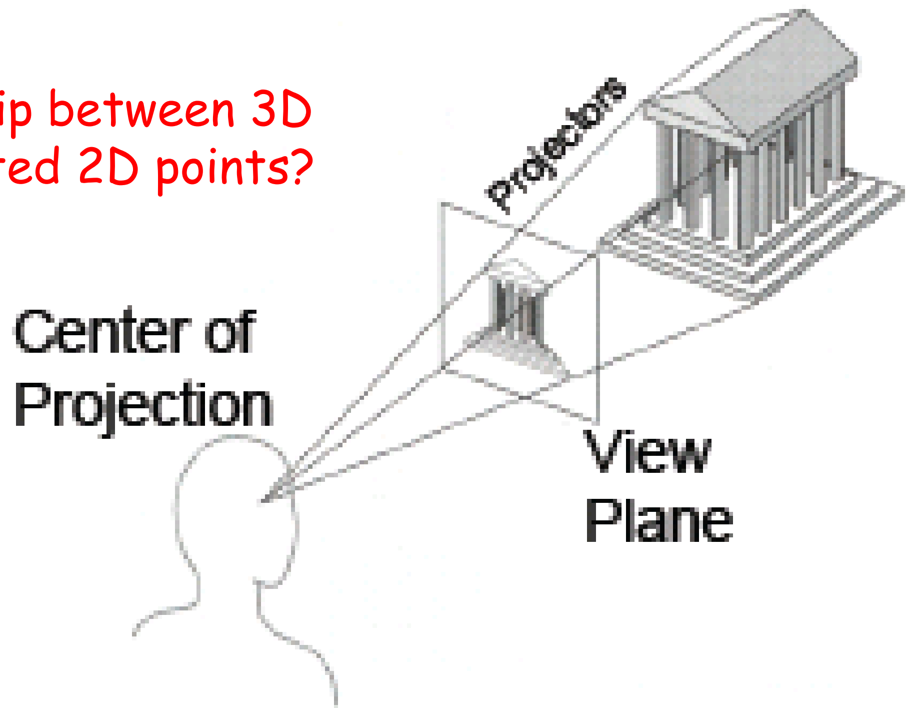
- Maps points onto "view plane" along projectors emanating from "center of projection" (COP) (κέντρο προβολής / σημείο αναφοράς της προβολής)



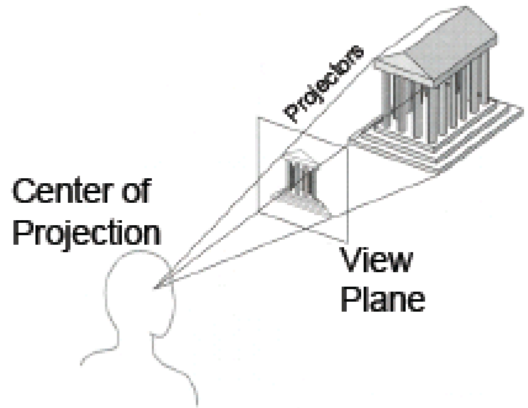
Perspective Projection

- Maps points onto "view plane" along projectors emanating from "center of projection" (COP)

What's relationship between 3D points and projected 2D points?



Camera->Screen



- Remember: Object->camera->screen

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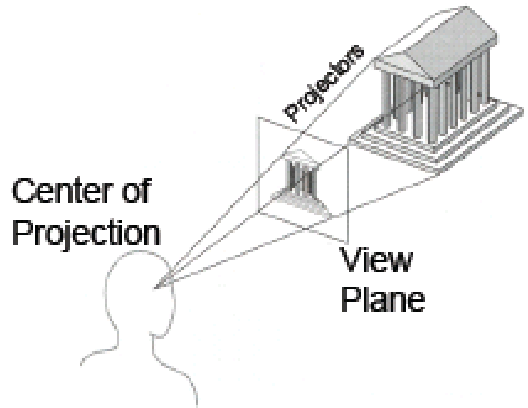

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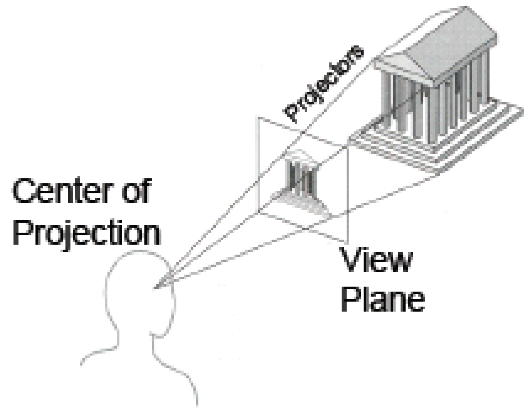

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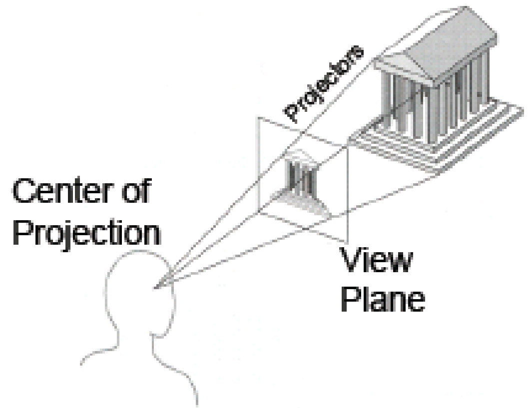

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- Its x and y axes is parallel to the x and y axes of eye coordinate system

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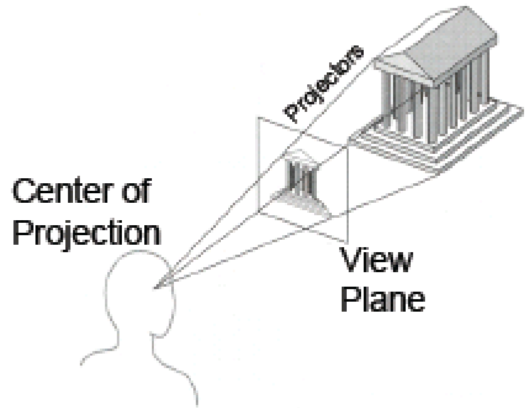
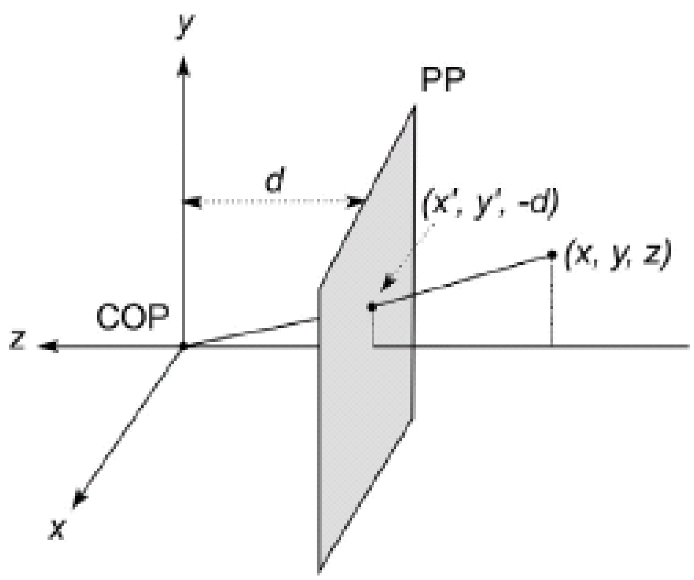


- Remember: Object->camera->screen
- Screen is z=-d plane for some constant d
- Coordinates of origin of screen is (0,0,-d)
- Its x and y axes is parallel to the x and y axes of eye coordinate system
- All these coordinates are in camera space now

Camera->Screen

- Consider the projection of a point on the camera plane



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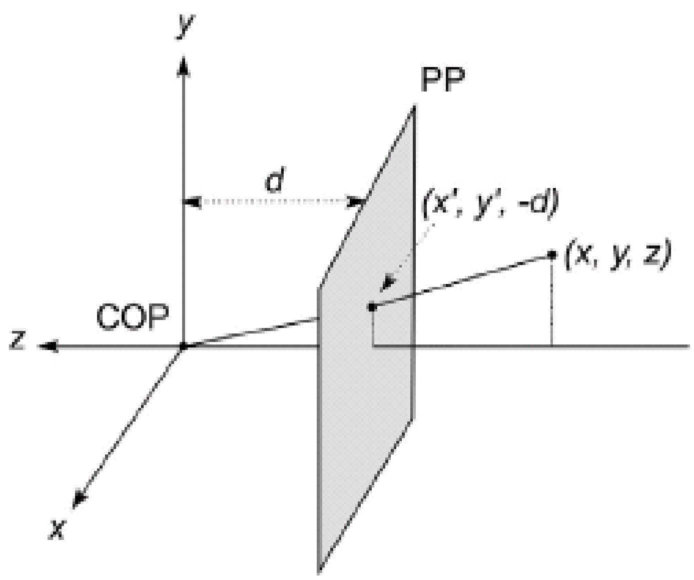

By similar triangles, we can compute how much the x and y coordinates are scaled

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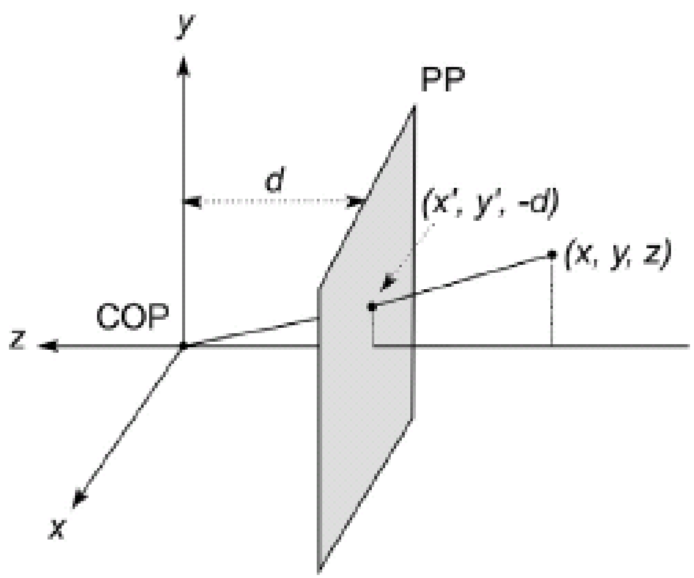

$$\frac{x}{z} = \frac{x'}{-d} \Rightarrow x' = \frac{-dx}{z}$$

$$\frac{y}{z} = \frac{y'}{-d} \Rightarrow y' = \frac{-dy}{z}$$

By similar triangles, we can compute how much the x and y coordinates are scaled

Homogeneous Point Revisited

- Remember how we said 2D/3D geometric transformations work with the last coordinate always set to one
- What happens if the coordinate is not one
- We divide all coordinates by *w*:

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \rightarrow \begin{bmatrix} x/w \\ y/w \\ z/w \\ 1 \end{bmatrix}$$

If w=1, nothing happens
Sometimes, we call this division step the "perspective divide"

The Perspective Matrix

- Now we can rewrite perspective projection equation as a matrix equation

$$\frac{x}{z} = \frac{x'}{-d} \Rightarrow x' = \frac{-dx}{z}$$

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$$\Rightarrow$$

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

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$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} x \\ y \\ -\frac{z}{d} \end{pmatrix}}$$

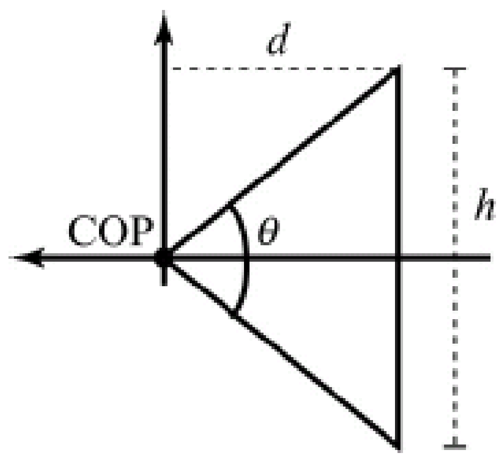
After the division by w, we have

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} -dx/z \\ -dy/z \\ 1 \end{pmatrix}$$

Note that this is not a linear transformation!!

Viewing Angle

- An alternative to specifying the distance between COP and PP is to specify viewing angle:



Given the height of the image *h* and θ , what is *d*?

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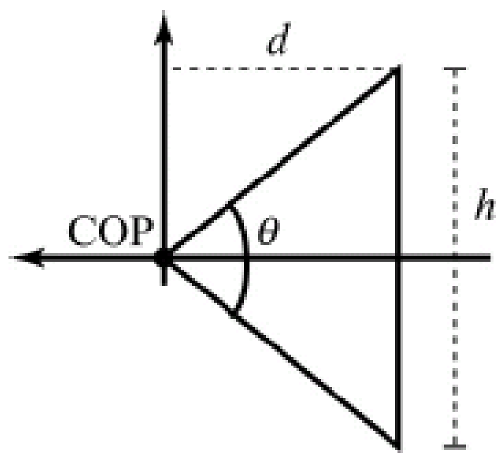

Given the height of the image *h* and θ , what is *d*?

$$\tan\frac{\theta}{2} = \frac{h}{2d}$$

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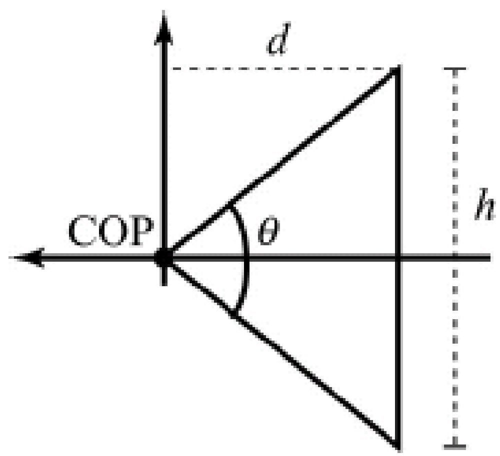

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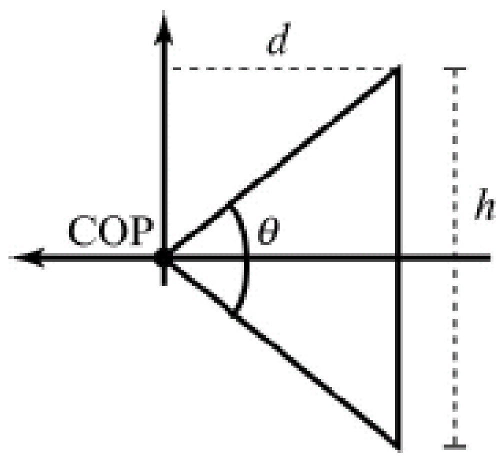

Given the height of the image *h* and θ , what is *d*?

$$\tan\frac{\theta}{2}=\frac{h}{2d} \quad \Rightarrow d=\frac{h\cdot\cot\frac{\theta}{2}}{2} \quad \Rightarrow d=\frac{w\cdot\cot\frac{\theta}{2}}{2\cdot aspect}$$

where aspect= height/width of the view plane

Viewing Angle

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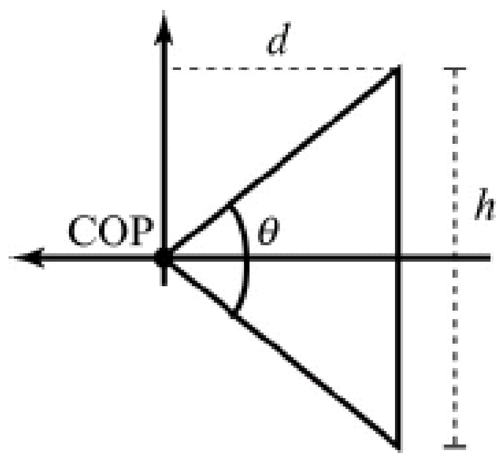

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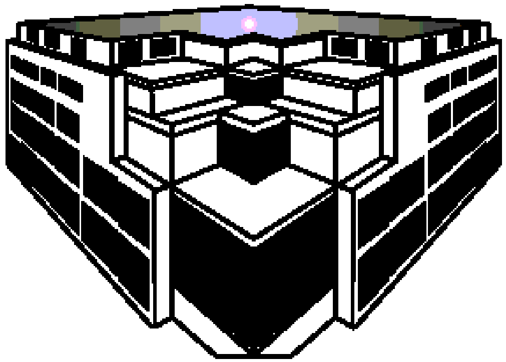
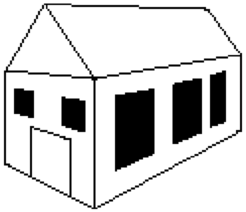
What happens to *d* as θ increases (keep *d* constant)?

Perspective Projections



- Used for:
  - fine art
  - Human visual system…
- Pros:
  - gives a realistic view and feeling for 3D form of object
- Cons:
  - does not preserve shape of object or scale (except where object intersects projection plane)
- Different from a parallel projection because
  - parallel lines not parallel to the projection plane converge
  - size of object is diminished with distance
  - foreshortening is not uniform
- Two understandings: **Vanishing Point(σημείο διαφυγής) and View Point (σημείο θέασης)**
- There are also oblique perspective projections (same idea as parallel oblique).



If we were viewing this scene using parallel projection, the tracks would not converge

Vanishing Points - Σημεία διαφυγής

- What happens to parallel lines they are not parallel to the projection plane?

Vanishing Points

- What happens to parallel lines they are not parallel to the projection plane?

- The equation of the line:

$$l = \vec{p} + t\vec{v} = \begin{pmatrix} p_x \\ p_y \\ p_z \\ 1 \end{pmatrix} + t \begin{pmatrix} v_x \\ v_y \\ v_z \\ 0 \end{pmatrix}$$

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- After perspective transformation, we have

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} p_x + tv_x \\ p_y + tv_y \\ -(p_z + tv_z)/d \end{pmatrix}$$

Vanishing Points (cont.)

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$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} p_x + tv_x \\ p_y + tv_y \\ -(p_z + tv_z)/d \end{pmatrix}$$

- Divided by w:

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} -\dfrac{p_x + tv_x}{p_z + tv_z} d \\ -\dfrac{p_y + tv_y}{p_z + tv_z} d \\ 1 \end{pmatrix}$$

- Letting t go to infinity:

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} -\dfrac{v_x}{v_z} d \\ -\dfrac{v_y}{v_z} d \\ 1 \end{pmatrix}$$

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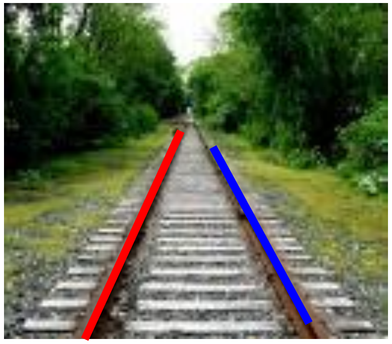

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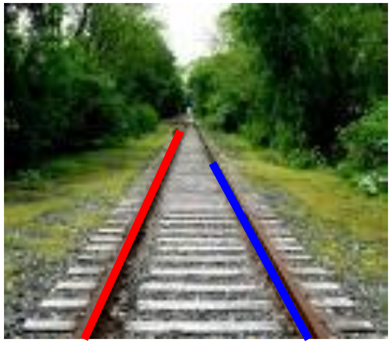

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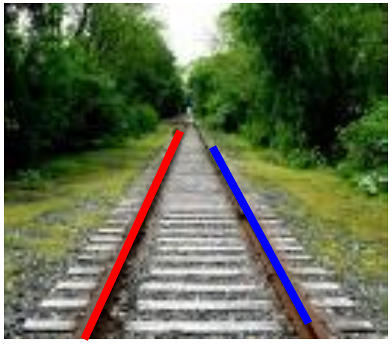

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Same vanishing point!

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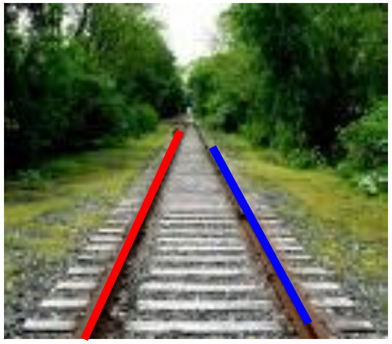

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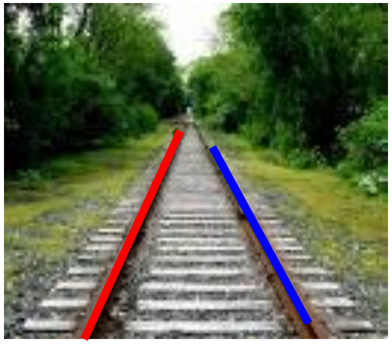

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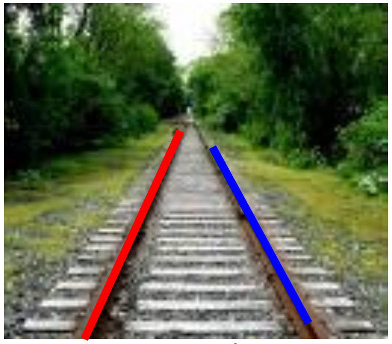

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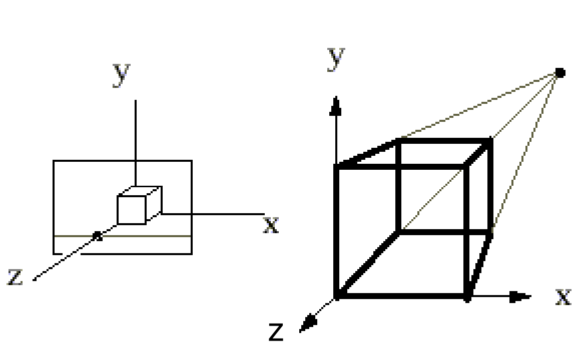
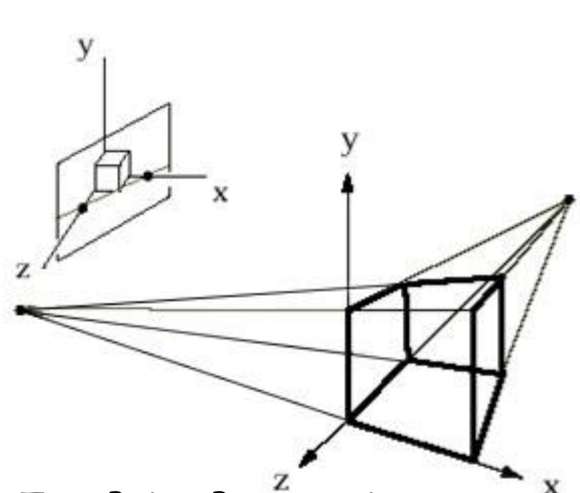
- How many vanishing points are there? ∞

Vanishing Points

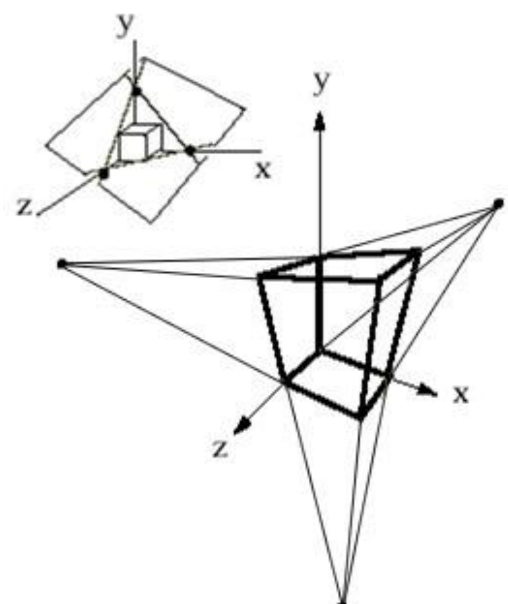
- Lines extending from edges converge to common vanishing point(s)
- For right-angled forms whose face normals are perpendicular to the *x*, *y*, *z* coordinate axes, number of vanishing points = number of principal coordinate axes intersected by projection plane



One Point Perspective
(*z*-axis vanishing point)



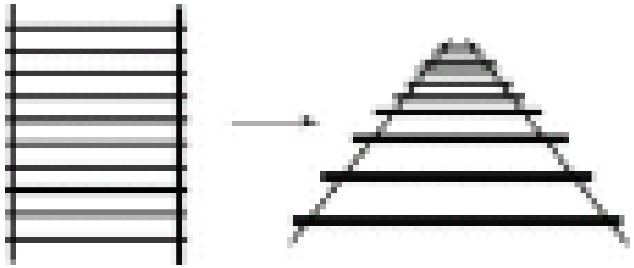
Two Point Perspective
(*z*, and *x*-axis vanishing points)



Three Point Perspective
(*z*, *x*, and *y*-axis vanishing points)

Properties of Perspective Proj.

- Perspective projection is an example projective transformation
  - lines maps to lines
  - parallel lines do not necessary remain parallel
  - ratios are not preserved



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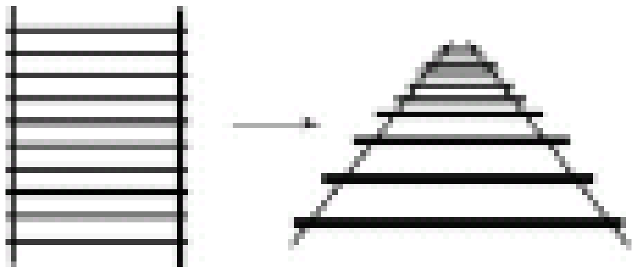

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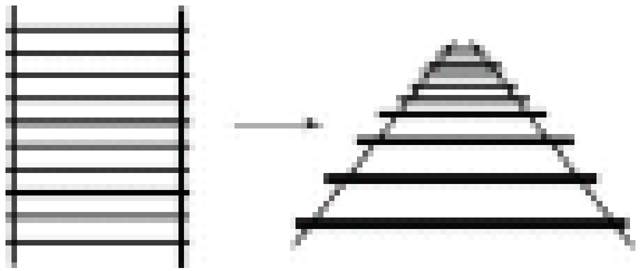
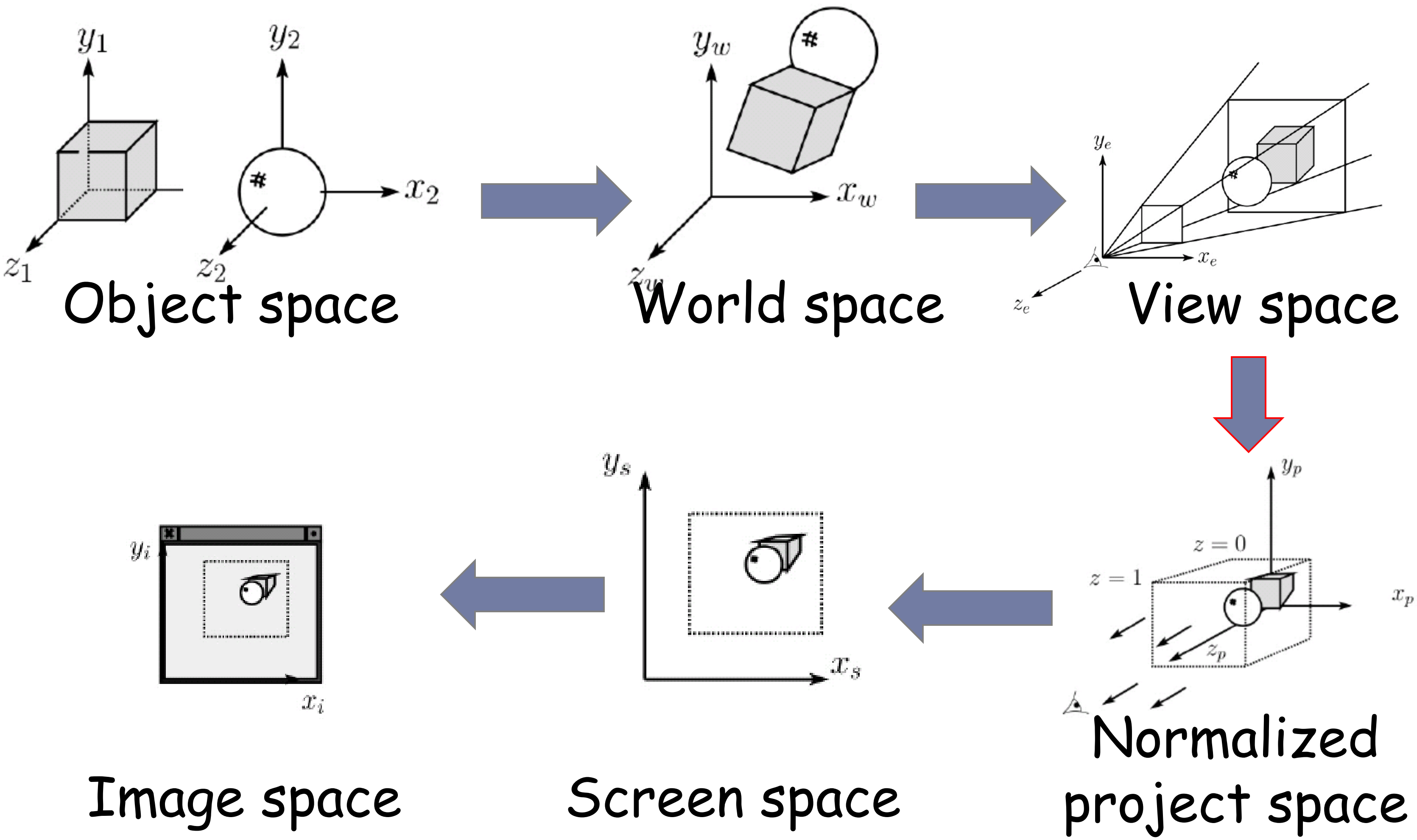


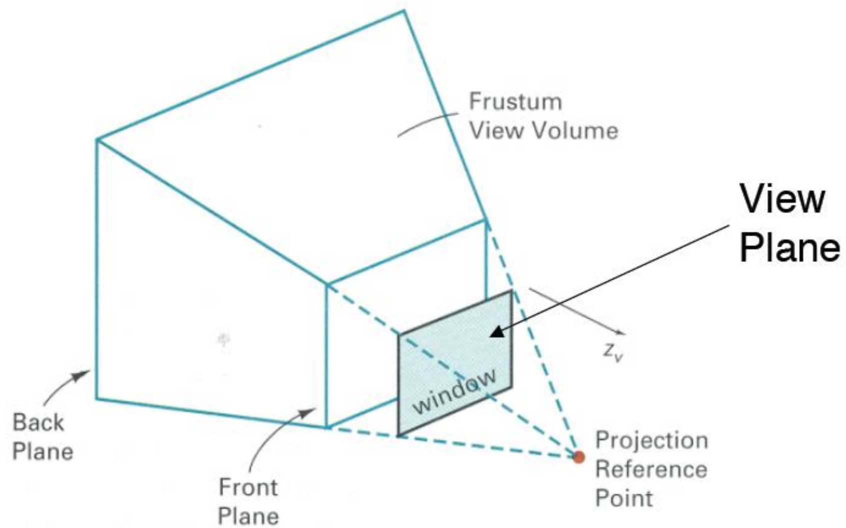
- One of advantages of perspective projection is that size varies inversely proportional to the distance → looks realistic
- We can not judge distances exactly as we can with parallel projection

3D Geometry Pipeline



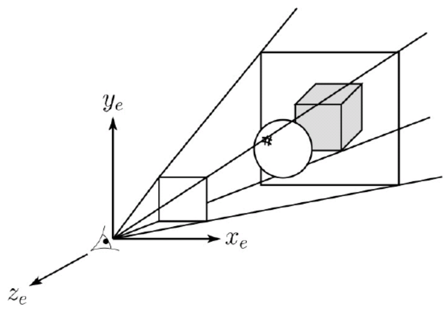
Perspective Projection Volume

- The center of projection and the portion of projection plane that map to the final image form an infinite pyramid. The sides of pyramid called clipping planes

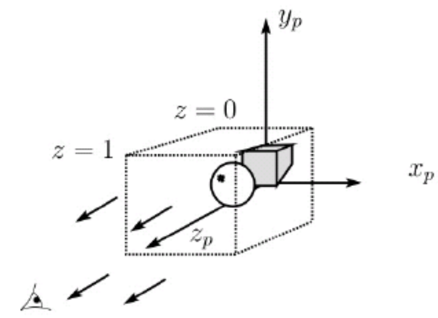


- Additional clipping planes are inserted to restrict the range of depths

Normalized Perspective-Projection



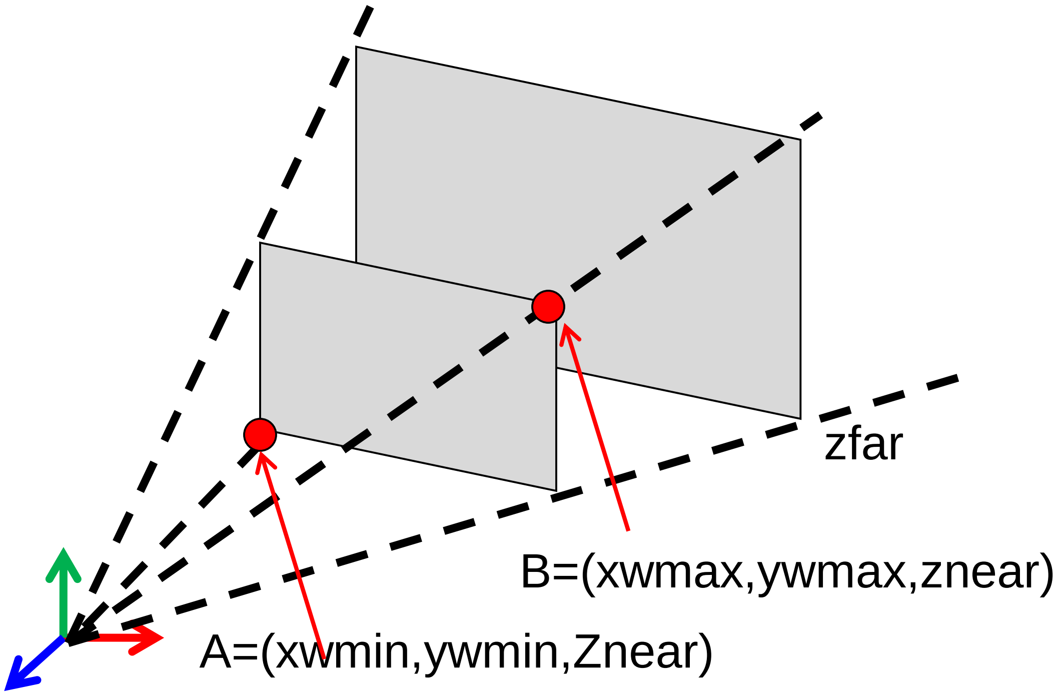
View space



Normalized project space

glFrustum(xwmin, xwmax,ywmin, ywmax ,dnear,dfar)

Normalized Perspective-Projection



View space

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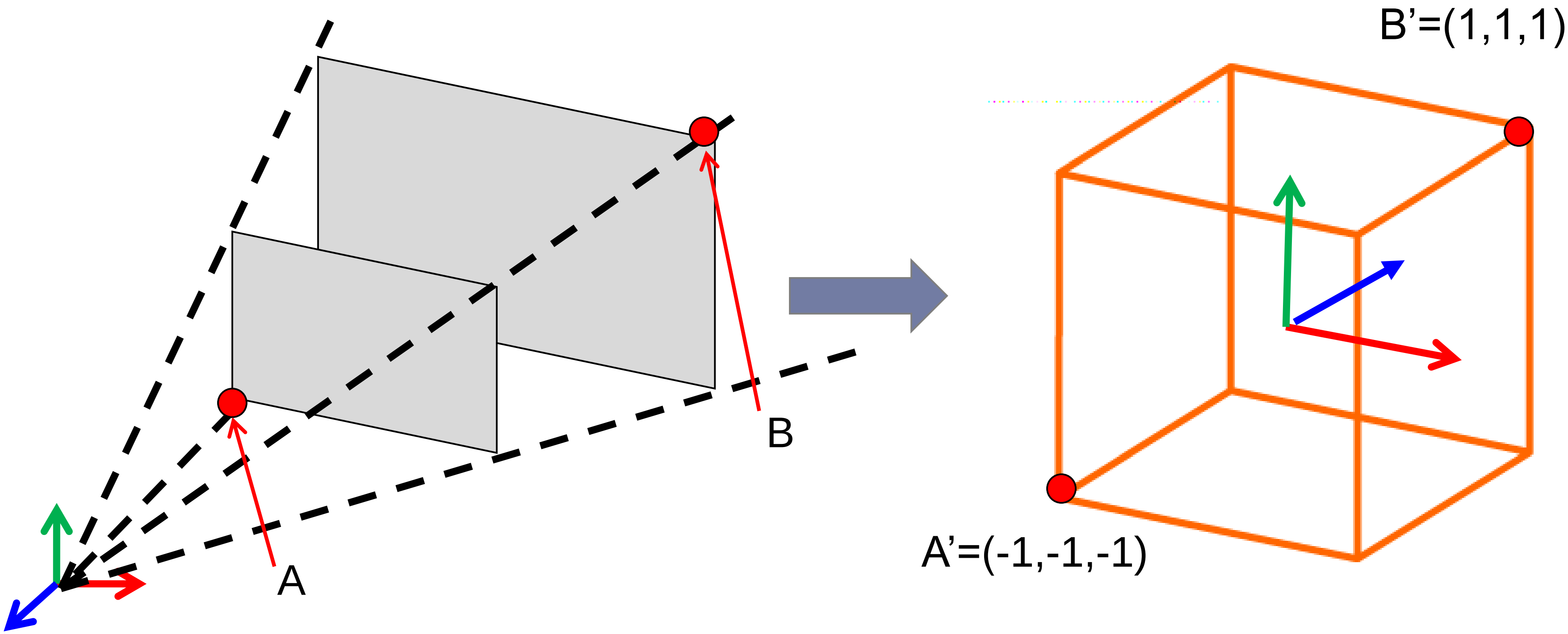
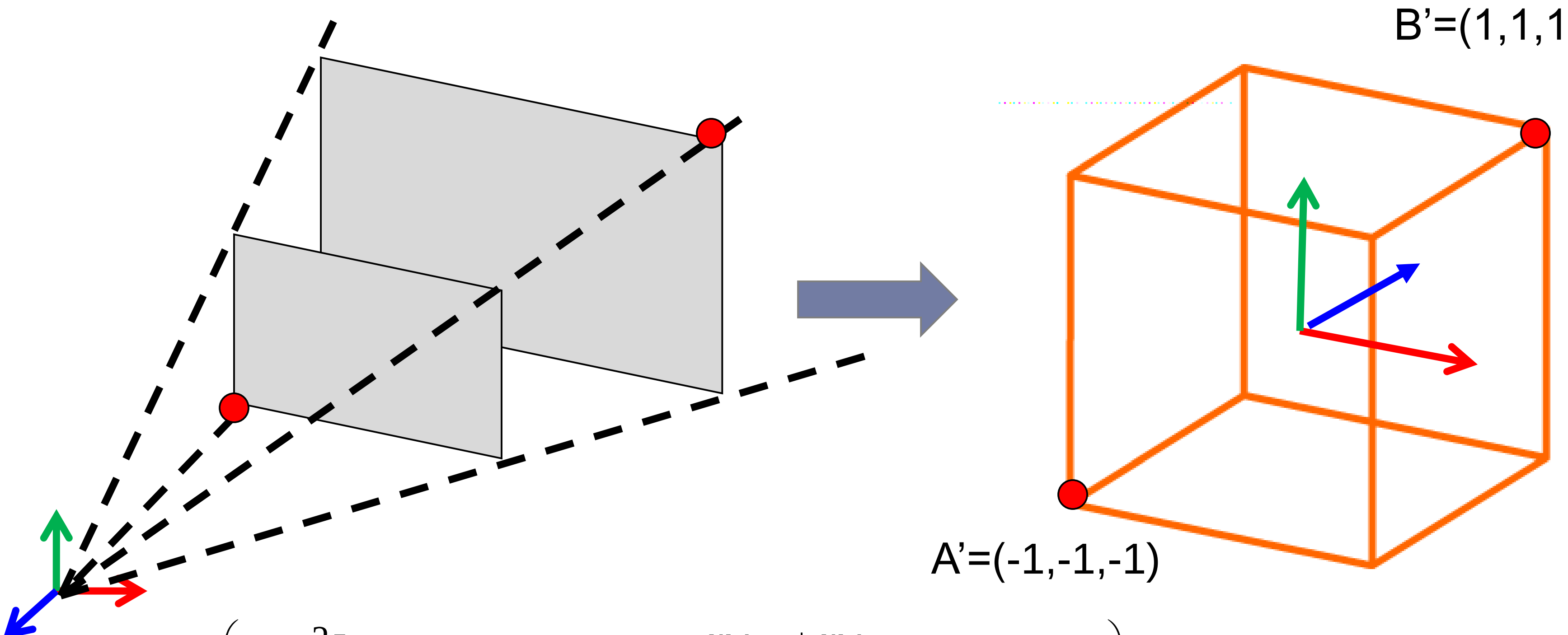


View space
(right-handed)

Normalized project space
(left-handed)

A maps to A', B maps to B'
Keep the directions of x and y axes!

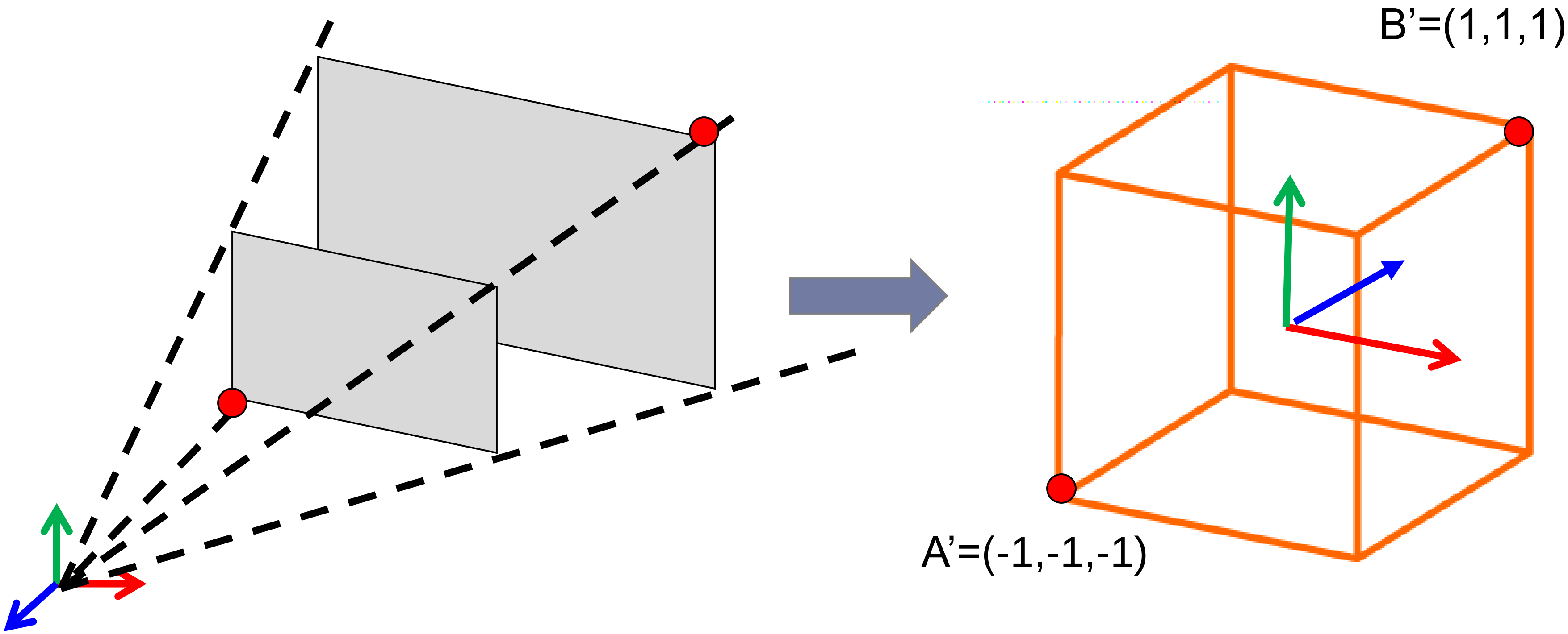
Normalized Perspective-Projection



$$T = \begin{pmatrix} \frac{-2z_{near}}{xw_{\max} - xw_{\min}} & 0 & \frac{xw_{\max} + xw_{\min}}{xw_{\max} - xw_{\min}} & 0 \\ 0 & \frac{-2z_{near}}{yw_{\max} - yw_{\min}} & \frac{yw_{\max} + yw_{\min}}{yw_{\max} - yw_{\min}} & 0 \\ 0 & 0 & \frac{z_{near} + z_{far}}{z_{near} - z_{far}} & -\frac{2z_{near} z_{far}}{z_{near} - z_{far}} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

H&B equation (7.40)

Normalized Perspective-Projection



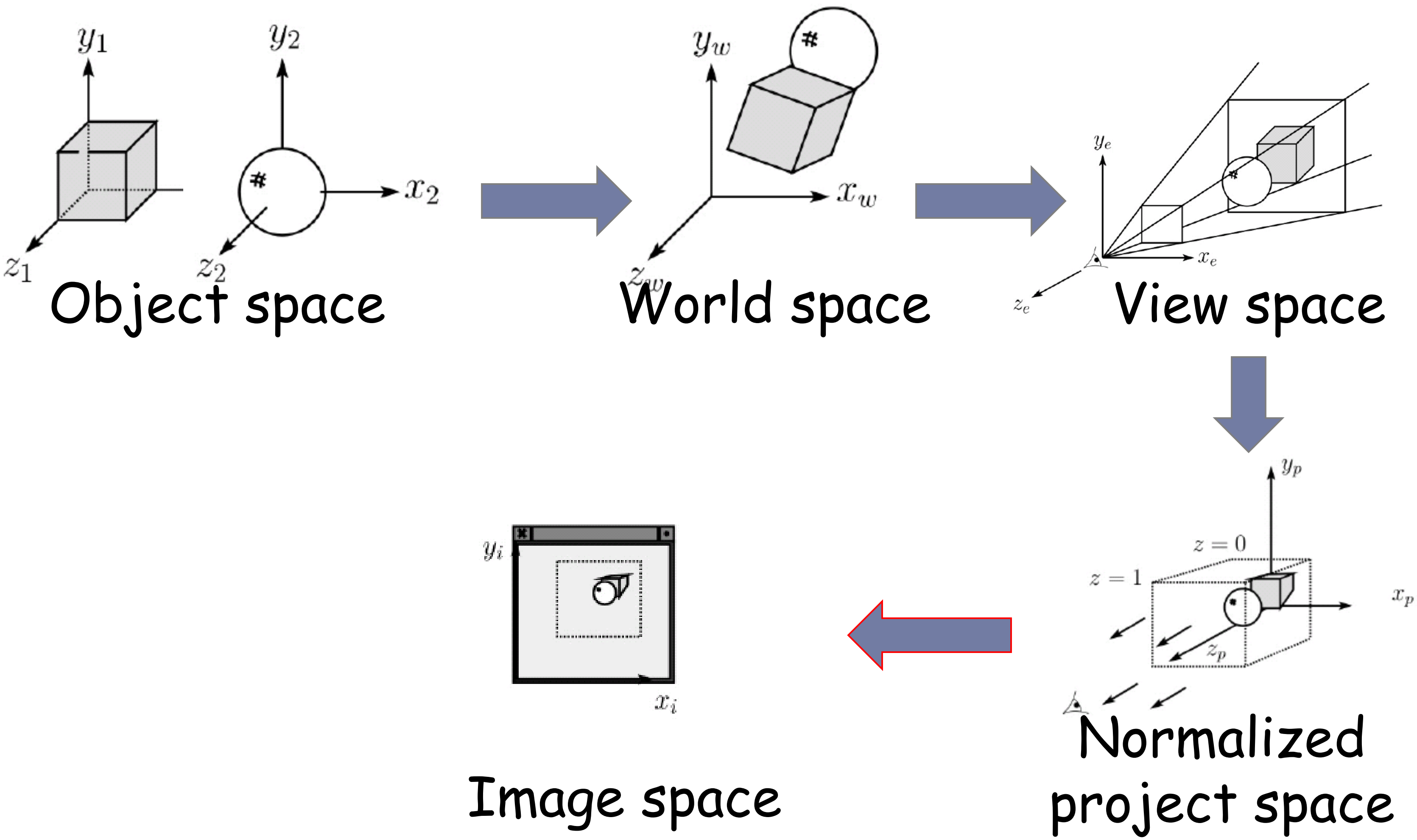
View space

Normalized project space

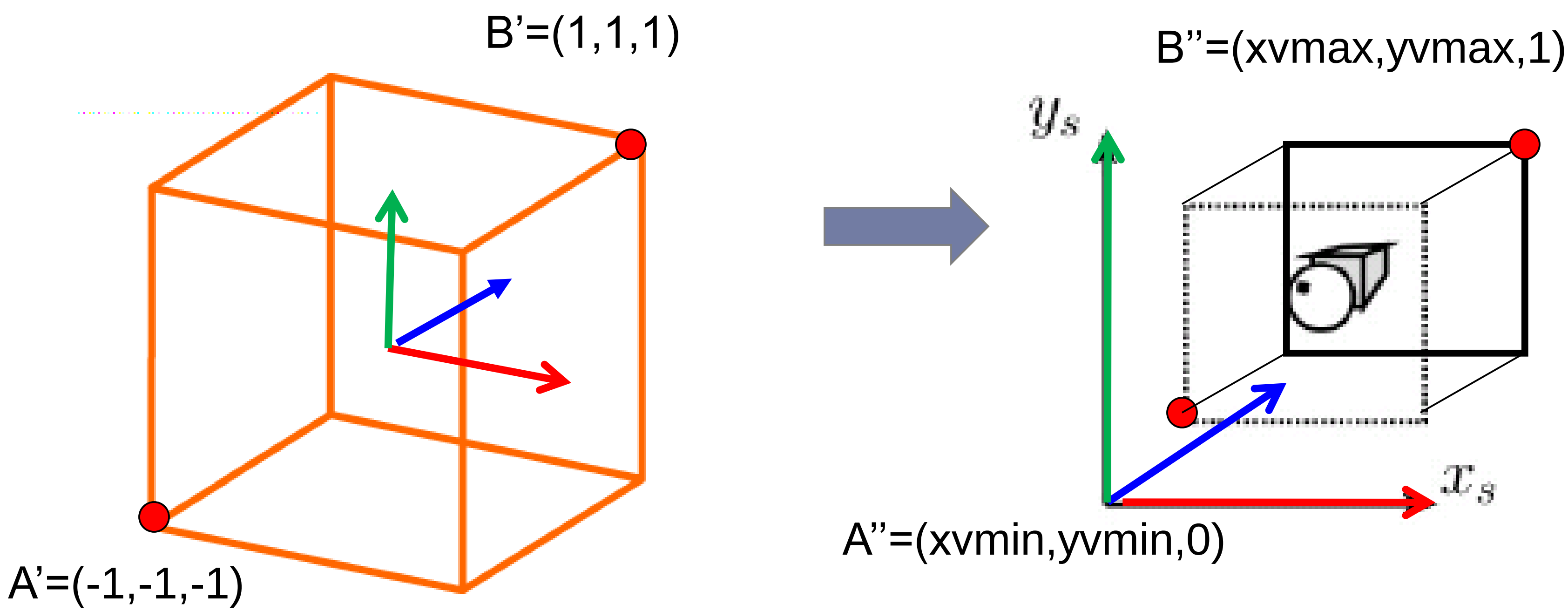
glFrustum(xwmin, xwmax,ywmin, ywmax ,dnear,dfar)

$$d_{near} = -z_{near} \qquad d_{far} = -z_{far}$$

3D Geometry Pipeline



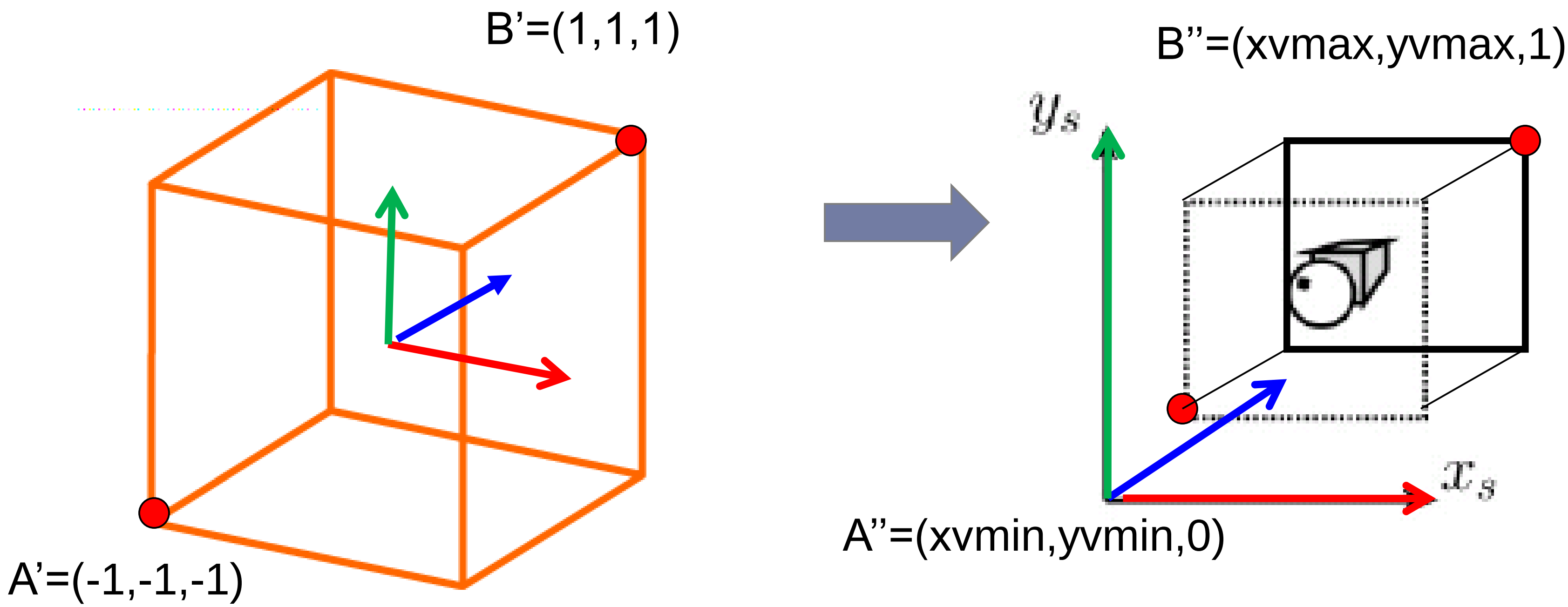
Viewport Transformation



Normalized project space

Image space

Viewport Transformation



Normalized project space

Image space

$$T = \begin{pmatrix} \frac{xv_{max} - xv_{min}}{2} & 0 & 0 & \frac{xv_{max} + xv_{min}}{2} \\ 0 & \frac{yv_{max} - yv_{min}}{2} & 0 & \frac{yv_{max} + yv_{min}}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

H&B equation (7.42)

Viewport Transformation

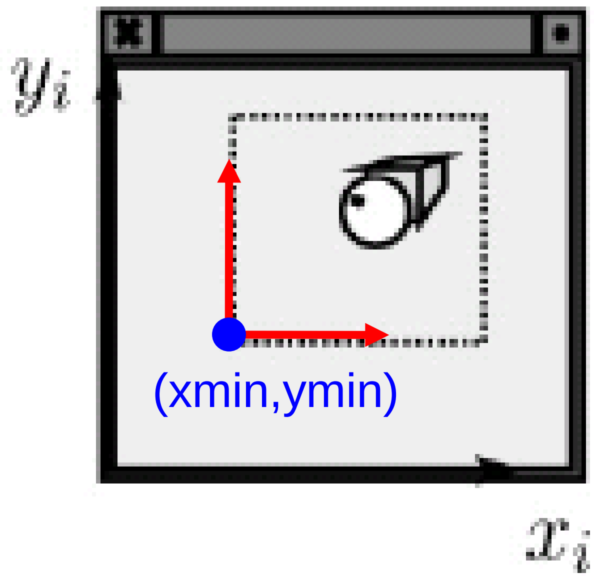
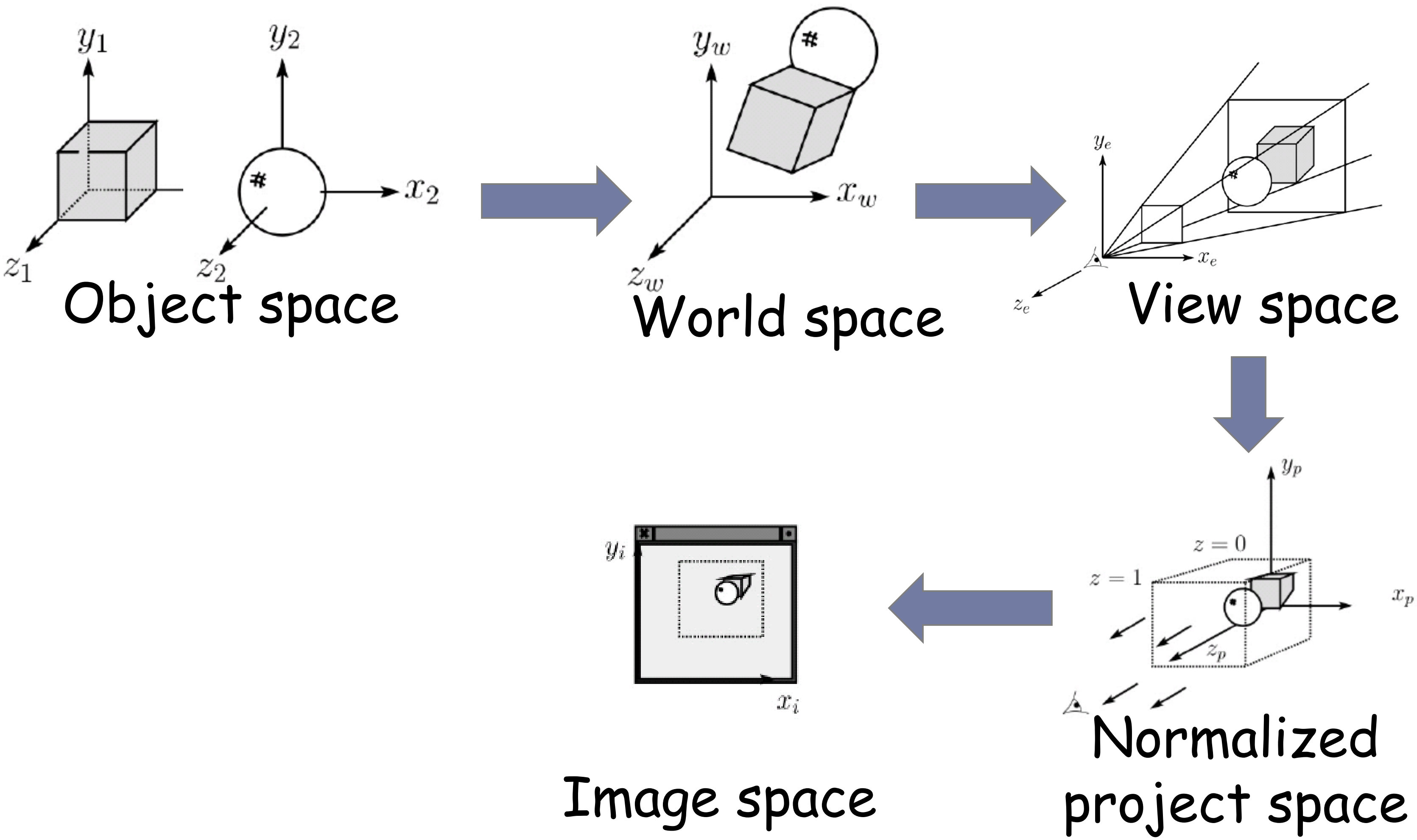


Image space → Image space

gluViewport(xmin, ymin, width, height)

Summary: 3D Geometry Pipeline



Summary : Object Coordinate to Device Coordinate

- Take your representation (points) and transform it from Object Space to World Space (M_{wo})
- Take your World Space point and transform it to Camera Space (M_{cw})
- Perform the remapping and projection onto the image plane in Normalized Device Coordinates (M_{nd_p} M_{pc})
- Perform this set of transformations on each point of the polygonal object ($M = M_{nd\_p} M_{pc} M_{cw} M_{wo}$)

Ερωτήσεις

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